RJP Problems

Mathematica problems MM1, MM2, & MM3 are found under the Mathematical Summary document.

RJP-10:

Write a fortran program that computes the x and y coordinates of the points on a circle of arbitrary radius R (the value of which may be entered from the keyboard) centered at the origin. Use the polar equations x= R cos (theta) and y=R sin (theta) and step theta from 0 to 360 degrees, in N steps, where N is also entered from the keyboard. Print the values of x and y into an unformatted output file. Import that file into Excel and plot the results full-page. Also print the coordinates to another file that is a formatted table as explained in my online FORTRAN manual. The table should be centered on the page and be continued on any following pages with the statement " Table 1 Continued" and then the column headings again.

Submit the source code, the output table and the Excel graph on a date to be announced. The 3 pages are to be stapled together in the order listed. Do this for R=10 and N=90.

RJP-330. Solve the following set of equations for x, y, and z by the method of row reduction:

x - 5y = 14
2x +7z = 15
x -y + 3z = 9

RJP-331. Solve the following set of equations for x, y, and z by the method of row reduction:

2x + 3y -z = -3
x + y + z = 2
-x + y - 2z = 2

RJP-332. Solve the following set of equations using Cramer's Rule.

14 -x + 5y = 0
7z + 2x - 15 = 0
x - y + 3z = 9

RJP-333. For the matrices A and B given below, find the products AB and BA, if possible

$$\boldsymbol{A} = \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix} \qquad \boldsymbol{B} = \begin{pmatrix} -1 & 4 \\ 0 & 2 \end{pmatrix}$$

RJP-334. For the matrices A and B given below, find the products AB and BA, if possible.

$$\boldsymbol{A} = \begin{pmatrix} 2 & 3 & 1 & -4 \\ 2 & 1 & 0 & 5 \end{pmatrix} \qquad \boldsymbol{B} = \begin{pmatrix} 2 & 4 \\ 1 & -1 \\ 3 & -1 \end{pmatrix}$$

RJP-335. Find AA^T for the matrix A in RJP-34

See the document "SUPPLEMENTARY NOTES" regarding vector components and rotation of axes. The general rule is this: To find the component of a vector along any direction, find the dot product of that vector with a unit vector along the direction. Recall that i, j, and k are unit vectors along x, y, and z.

RJP-340. Given r = 12, θ = 40°, and φ = 50°, find r′ for a rotation around the y-axis thru angle ρ = 60°. First draw a diagram showing the vector r in a 3D diagram of spherical coordinates and label the components of the vector.

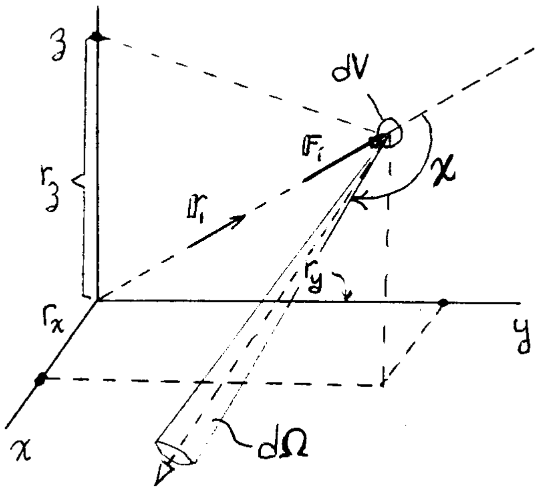
RJP-341. Given the vector **V** = 7**i** +9**j** +4**k**, find $V_{x'} + V_{y'} + V_{z'}$ in a frame that is rotated relative to the original frame by and angle θ = 20° around the z-axis. Once you have found the numerical values for $V_{x'}$, $V_{y'}$, and $V_{z'}$, prove the invariance of V.

See supplementary notes, now a separate document on my web page, regarding unit vectors and directional cosines.

RJP-346: In a given frame of reference, a space probe has a velocity **V** = -5 **i** +8 **j** +4**k**, when at the point x=10, y=8, z=10. Find the component of V along the direction from the probe's location towards the point x=4, y=12, z=2.

RJP-350 **A problem in atomic physics:**

Electron scattering of electromagnetic radiation:



Take the origin as a point source radiating isotropically a flux which is scattered by a small volume, dV, of electrons located at the point x,y,z=10,14,12. The amount of power scattered by the electrons into a solid angle, dΩ, is given by

$$dP = (3/16\pi)F_i N_e \sigma_e(1+\cos^2\chi)\,d\Omega\, dV,$$

where F_i is the flux (J/m²/sec) incident on the scattering volume element of electrons, dV; σ_e is the electron cross-section (6.65×10^{-29} m²), and χ is the scattering angle. The latter is defined as the angle between the original direction of F_i and any other direction. If F_i = 5.0 watts/m², $N_e = 3 \times 10^{12}$ m^{-3}, dV= 2.0×10^{-5} m³, calculate the value of dP/dΩ scattered in the x-direction.

RJP-360: Find the value of the magnetic field B (in teslas) necessary for electrons to produce synchrotron radiation with a wavelength of 550 nm. Use SI units.

RJP-500. Find the electric potential, PHI (a scalar field), at a point x=20m, y=20m, z=10m due to a uniform circular distribution of electric charges centered at the origin and of radius 10m. The total electric charge of the circular loop is 20 millicoulombs. There is no way to solve this problem other than by numerical integration. Think FORTRAN and use the variable names indicated in the table below. The answer is in volts (V). Format an output table of the data in columns as shown:

TABLE 1

Results of the Numerical Integration of the
Electric Potential for a Ring of Charge

N	X	Y	S	DELPHI(V)	PHISUM(V)
1	10.00	00.00	22.01	82345.0	82345.0

Etc.

Center on the page. Use N=90, as in RJP-10. N is the number of the charge element. X and Y are the coordinates of the charge element. S is the distance from the charge element to the field point. The numbers in the 5^{th} column are the differential contributions of each element of charge to the total potential. The last column is the running sum of the differential contributions so that the last number in the last column should be the answer.

RJP-515. Consider the region bounded by the parabolic cylinder $z= 4-x^2$ and the planes x=0, y=0, y=6, and z=0. Sketch the geometry of the region relative to the x, y, and z axes.

A. Find the volume of the above region.

B. Find the centroid of the above region.

RJP-521. Work out the details of the triple integral for the moment inertia around the x-axis as set up in Example 2(b) on page 254 of Boas. Hint: break the integral of (y^2+z^2) dy into two parts.

RJP-526. The region of our Galaxy in the vicinity of the Sun may be represented as a disk, in which the density of stars in the central xy plane (where the Sun is located) is constant but varies as:

$\rho = \rho_o \exp\left(-\frac{|z|}{\beta}\right)$ perpendicular to the central plane. Calculate the number of stars in the vicinity of the Sun out to 100 parsecs in all directions, if ρ_o = 1/9 per cubic parsec and β = 500 parsecs. Notice that the argument of the exponential in the density function is dimensionless. Use rectangular coordinates.

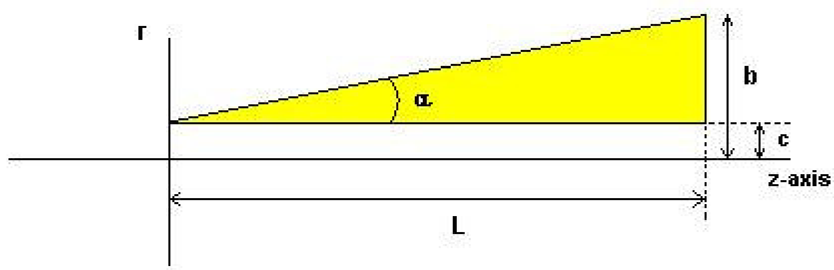
RJP-410 The van der Waal equation of state for a gas is (P+b)(v-a) = nRT, where b and a are atomic constants. Find $\left(\frac{\partial P}{\partial v}\right)_T$. See the Supplementary Notes.

RJP-531: Find the Jacobian, J(r, s, t) for the transformation from xyz-space to rst-space when

$$x = \frac{5r^2 + 5t^2}{10} - s^2; \quad y = \frac{5s^2 - 5t^2}{10} + r^2; \quad z = 10r^2$$

RJP-545:

Find the moment of inertia about the z axis of the solid of revolution formed by a right triangle offset from the z-axis by distance c as depicted below. The length of the triangle is L.



RJP-645: Prove that the integral of **D**, arising from a point charge, over the surface of an arbitrarily shaped closed surface that does not enclose any charge, is zero. Hint: Use solid angles.

RJP-685: Use Gauss's Law to find the electric field **E** at an arbitrary distance r from the center axis of an infinitely long cylinder of uniform charge density rho (coulombs per cubic centimeter). Present all simplifying arguments clearly and completely, taking into account the vector nature of the field.

RJP-690: Verify Stokes' Theorem for the magnetic intensity **B**= 2xy **i** + x^2 **j** for the region in the xy-plane bounded by the triangle with corners at (0,0), (a,0), and (a,a).

RJP-693 Verify Stokes's Theorem for **V** = 3y **i** -xy **j**+ yz^2 **k** over the surface of the parabolid $2z=x^2 + y^2$ bounded by z=2. Draw a diagram approximately 3 x 3 inches showing the surface, the bounding curve and normal vectors needed. Change to cylindrical coordinates.

RJP-708 Use trigonometric identities to evaluate <sin mx cosnx> over the interval 0 to 2π.

RJP-721 Expand the function given by equation (5.11) on page 353 in a complex Fourier Series.