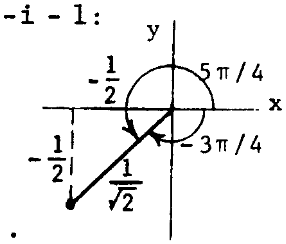
Section 5

2-5 2. See text Example 2, Section 5A. We multiply numerator and denominator by the conjugate of $i - 1$ which is $-i - 1$:



$$\frac{1}{i-1} \cdot \frac{-i-1}{-i-1} = \frac{-i-1}{2} = -\frac{1}{2} - \frac{1}{2} i.$$

We plot the point $(-1/2, -1/2)$, and find

$r = \sqrt{(-1/2)^2 + (-1/2)^2} = \sqrt{1/2}$, $\theta = -3\pi/4$ (or $5\pi/4$).

The 5 ways of labeling the point are $(-1/2, -1/2)$, $-1/2 - i/2$, $(1/\sqrt{2}, -3\pi/4)$, $\frac{1}{\sqrt{2}}\left(\cos\frac{3\pi}{4} - i \sin\frac{3\pi}{4}\right)$, $\frac{1}{\sqrt{2}} e^{-3\pi i/4}$.

We could also do this problem by writing $i - 1$ in polar form. We sketch (or picture mentally), the point $(-1,1)$ (see text Figure 9.6) and find $i - 1 = \sqrt{2}e^{3\pi i/4}$. Then

$$\frac{1}{i-1} = \frac{1}{\sqrt{2}e^{3\pi i/4}} = \frac{1}{\sqrt{2}} e^{-3\pi i/4}$$ as above.

2-5 6. Using polar coordinates, we write $1 + i = \sqrt{2}e^{i\pi/4}$ (visualize the sketch or see Problem 4.1) and $1 - i = \sqrt{2}e^{-i\pi/4}$. Then

$$\left(\frac{1+i}{1-i}\right)^2 = \left(\frac{\sqrt{2}e^{i\pi/4}}{\sqrt{2}e^{-i\pi/4}}\right)^2 = \left(e^{i\pi/2}\right)^2 = e^{i\pi} = -1.$$

The ways of labeling this point are given in Problem 4.7.