

B. 3-6.22

$$\text{Given } \begin{cases} x-y+z=4 \\ 2x-y-z=-1 \\ 3x+2y+2z=5 \end{cases} \Rightarrow \underbrace{\begin{pmatrix} 1 & -1 & 1 \\ 2 & -1 & -1 \\ 3 & 2 & 2 \end{pmatrix}}_A \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_r = \underbrace{\begin{pmatrix} 4 \\ -1 \\ 5 \end{pmatrix}}_{r'}$$

$$\det A = 1(2+2) + 1(4+3) + 1(4-3) = 4 + 7 + 1 = 12$$

Cofactors matrix

$$C = \begin{pmatrix} \begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} \\ -\begin{vmatrix} -1 & 1 \\ 2 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} \\ \begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} 4 & -7 & 1 \\ 4 & -1 & -5 \\ 0 & 3 & 3 \end{pmatrix}$$

$$C^T = \begin{pmatrix} 4 & 4 & 0 \\ -7 & -1 & 3 \\ 1 & -5 & 3 \end{pmatrix}$$

$$A^{-1} = \frac{C^T}{\det A} = \frac{1}{12} \begin{pmatrix} 4 & 4 & 0 \\ -7 & -1 & 3 \\ 1 & -5 & 3 \end{pmatrix}$$

$$r = A^{-1}k = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{12} \begin{pmatrix} 4 & 4 & 0 \\ -7 & -1 & 3 \\ 1 & -5 & 3 \end{pmatrix} \begin{pmatrix} 4 \\ -1 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{12} \begin{pmatrix} 4 \cdot 4 + 4(-1) + 0 \\ -7 \cdot 4 + (-1)(-1) + 3 \cdot 5 \\ 4 \cdot 1 + (-5)(-1) + 3 \cdot 5 \end{pmatrix} = \frac{1}{12} \begin{pmatrix} 16 - 4 + 0 \\ -28 + 1 + 15 \\ 4 + 5 + 15 \end{pmatrix} = \frac{1}{12} \begin{pmatrix} 12 \\ -12 \\ 24 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$