**Boas 3-6.25** Two successive rotations in the xy-plane rather than two successive rotations of a vector. In this case, the rotatioal matrix is just the transpose of (6.14). That is $\mathbf{r}'' = C\mathbf{r}$

where
$$C = \begin{pmatrix} \cos(\theta+\varphi) & \sin(\theta+\varphi) \\ -\sin(\theta+\varphi) & \cos(\theta+\varphi) \end{pmatrix}$$

Then $\quad r'' = B(Ar) = BAr \qquad$ Assoc. Rule

Proof $\quad BA = C$ :

$$BA = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \overset{x}{\cos\varphi}\,\overset{y}{\cos\theta} - \overset{x}{\sin\varphi}\,\overset{y}{\sin\theta} & \overset{y}{\cos\varphi}\,\overset{x}{\sin\theta} + \overset{y}{\sin\varphi}\,\overset{x}{\cos\theta} \\ -\overset{x}{\sin\varphi}\,\overset{y}{\cos\theta} - \overset{x}{\cos\varphi}\,\overset{y}{\sin\theta} & -\sin\varphi\sin\theta + \cos\varphi\cos\theta \end{pmatrix}$$

Now use

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

where $x = \varphi$ and $\theta = y$

$$\therefore \quad BA = C$$