

Ques 3-9.3

GIVEN:

$$A = \begin{pmatrix} 1 & 0 & 5i \\ -2i & 2 & 0 \\ 1 & 1+i & 0 \end{pmatrix}$$

$$\det A = 5i[-2i(1+i) - 2] = 5i[-2i + 2 - 2] = -10i \cdot 2 = 10$$

Transpose $A' = \begin{pmatrix} 1 & -2i & 1 \\ 0 & 2 & 1+i \\ 5i & 0 & 0 \end{pmatrix} = A^T$

Complex Conjugate $A^* = \bar{A} = \begin{pmatrix} 1 & 0 & -5i \\ 2i & 2 & 0 \\ 1 & 1-i & 0 \end{pmatrix}$ transpose conjugate $A^\dagger = \begin{pmatrix} 1 & 2i & 1 \\ 0 & 2 & 1-i \\ -5i & 0 & 0 \end{pmatrix}$

Adjoint $(-1)^{ij} M_{ij} = C^T = \hat{A} = \begin{pmatrix} 0 & 0 & \begin{vmatrix} -2i & 2 \\ 1 & 1+i \end{vmatrix} \\ -\begin{vmatrix} 0 & 5i \\ 1+i & 0 \end{vmatrix} & \begin{vmatrix} 1 & 5i \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 1 & 1+i \end{vmatrix} \\ \begin{vmatrix} 0 & 5i \\ 2 & 0 \end{vmatrix} & -\begin{vmatrix} 1 & 5i \\ -2i & 0 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ -2i & 2 \end{vmatrix} \end{pmatrix}^T = \begin{pmatrix} 0 & 5i-5 & -10i \\ 0 & -5i & 10 \\ -2i & -1-i & 2 \end{pmatrix}$

$$A^{-1} = \frac{\hat{A}}{|A|} = \begin{pmatrix} 0 & \frac{1}{2}(i-1) & -i \\ 0 & -\frac{1}{2}i & 1 \\ -\frac{1}{2}i & -\frac{1}{10}(1+i) & \frac{1}{5} \end{pmatrix}$$

In general

$$AA^{-1} = A^{-1}A = U$$

B. 3-9.3 cont'd

$$A^{-1} = \begin{pmatrix} 0 & \frac{1}{2}(i-1) & -i \\ 0 & -\frac{1}{2}i & 1 \\ -\frac{1}{5}i & -\frac{1}{10}(i+1) & \frac{1}{5} \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 5i \\ -2i & 2 & 0 \\ 1 & 1+i & 0 \end{pmatrix}$$

$$A^{-1}A = \begin{pmatrix} 0 - i(i-1) - i & 0 + (i-1) - (i-1) & 0 + 0 + 0 \\ 0 + i^2 + 1 & 0 - i + 1 + i & 0 + 0 + 0 \\ -\frac{1}{5}i + \frac{1}{5}i(i+1) + \frac{1}{5} & 0 - \frac{1}{5}(1+i) + \frac{1}{5}(1+i) & -\frac{5i^2}{5} + 0 + 0 \end{pmatrix}$$

$$A^{-1}A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = U$$

$$\therefore A^{-1}A = U$$