3-11.13

To find the eigenvalues of the given matrix, we subtract μ from the elements of the main diagonal and set the resulting determinant equal to zero [see text equation (4.3) and the shaded box below it].

$$\begin{vmatrix} 2-\mu & 2 \\ 2 & -1-\mu \end{vmatrix} = 0.$$

Then we evaluate this determinant and solve the resulting equation (called the characteristic equation of the matrix).

$$\mu^2 - \mu - 6 = 0,$$

$$(\mu - 3)(\mu + 2) = 0, \qquad \mu = 3, \qquad \mu = -2.$$

Now an eigenvector (corresponding to each value of μ) is $\vec{i}x + \vec{j}y$ where (x,y) is a solution of the matrix equation

$$\begin{pmatrix} 2-\mu & 2 \\ 2 & -1-\mu \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0.$$

For $\mu = 3$, we find

$$\begin{pmatrix} -1 & 2 \\ 2 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0, \qquad \text{or} \qquad \begin{cases} -x + 2y = 0, \\ 2x - 4y = 0. \end{cases}$$

These two equations both represent the same straight line, say $x = 2y$. [Note that this kind of problem is self-checking; you must get equations representing one straight line corresponding to any (non-repeated) eigenvalue. If you don't, then hunt for your mistake.] A solution of $x = 2y$ is $x = 2$, $y = 1$, so an eigenvector corresponding to the eigenvalue $\mu = 3$ is $2\vec{i} + \vec{j}$ or (2,1). A corresponding unit eigenvector is $(2\vec{i} + \vec{j})/\sqrt{5}$. For $\mu = -2$, we find

$$\begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0 \qquad \text{or} \qquad 2x + y = 0.$$

A solution of this equation is $x = 1$, $y = -2$, so an eigenvector corresponding to $\mu = -2$ is (1,-2) and the corresponding unit eigenvector is $(1,-2)/\sqrt{5}$.