3-11.2

2. To show that $\vec{r}_1$ and $\vec{r}_2$ are perpendicular, we find

$$\vec{r}_1 \cdot \vec{r}_2 = \left(\frac{1}{\sqrt{5}}\right)\left(\frac{-2}{\sqrt{5}}\right) + \left(\frac{2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{5}}\right) = 0.$$

To verify that C is orthogonal, that is, that $C^T = C^{-1}$, we show that CC^T = unit matrix:

$$CC^T = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \frac{1}{\sqrt{5}}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

X 7. Let C be an n by n matrix whose columns are the components of n mutually perpendicular unit vectors $\vec{r}_1, \vec{r}_2, \cdots, \vec{r}_n$. Then

$$C^T C = \begin{pmatrix} x_1 & y_1 & \cdots \\ x_2 & y_2 & \cdots \\ \vdots & \vdots & \\ x_n & y_n & \cdots \end{pmatrix} \begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ y_1 & y_2 & \cdots & y_n \\ \vdots & \vdots & & \vdots \end{pmatrix}$$

$$= \begin{pmatrix} \vec{r}_1\cdot\vec{r}_1 & \vec{r}_1\cdot\vec{r}_2 & \cdots & \vec{r}_1\cdot\vec{r}_n \\ \vec{r}_2\cdot\vec{r}_1 & \vec{r}_2\cdot\vec{r}_2 & \cdots & \vec{r}_2\cdot\vec{r}_n \\ \vdots & \vdots & & \vdots \\ \vec{r}_n\cdot\vec{r}_1 & \vec{r}_n\cdot\vec{r}_2 & \cdots & \vec{r}_n\cdot\vec{r}_n \end{pmatrix} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

so $C^T = C^{-1}$. Thus C is an orthogonal matrix.