

6-11.4

Stokes Theorem says  $\int \nabla \times \mathbf{v} \cdot \mathbf{n} \, d\sigma = \oint_C \mathbf{v} \cdot d\mathbf{r}$  for any surface bounded by closed path  $C$ .

Hence,  $C$  for the surface given lies in the  $x, y$  plane and so we can substitute the triangular area in the  $x, y$  plane for the given surface. The plane

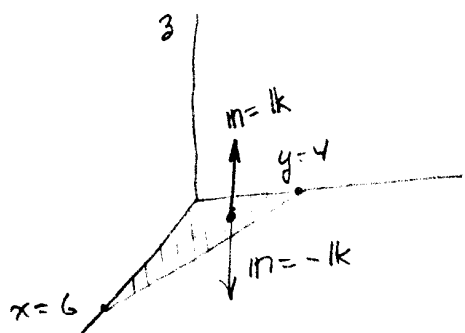
$$2x + 3y + 4z = 12$$

intercepts the  $x$  axis at  $x=6$ , & the  $y$  axis at  $y=4$

Now  $\nabla \times \mathbf{v} = \nabla \times (y\mathbf{i} + 2z\mathbf{j}) = -\mathbf{k}$

Hence  $(\nabla \times \mathbf{v}) \cdot \mathbf{n} = -\mathbf{k} \cdot \mathbf{k} = -1$

$$\begin{aligned} \int (\nabla \times \mathbf{v}) \cdot \mathbf{n} \, d\sigma &= \int (-1) \, d\sigma = -(\text{area of triangle}) \\ &= -\frac{1}{2} \cdot 6 \cdot 4 = -12 \end{aligned}$$



Note: Unlike the divergence theorem, where  $\mathbf{n}$  must be the outward normal of the surface enclosing volume  $\tau$ , Stokes Theorem is valid for all surfaces bounded by  $C$  that have the same sense of

concavity. In this case, the triangular area in the  $x, y$  plane has no concavity but we choose  $\mathbf{k}$  rather than  $-\mathbf{k}$ , since this is in the sense of curvature of the originally given surface.