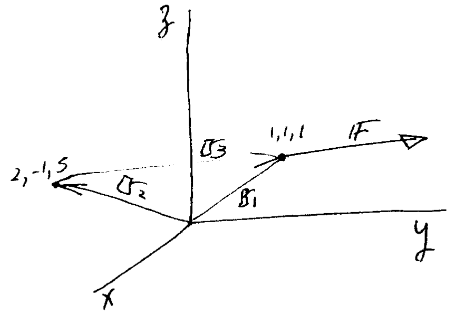
$\boxed{6\text{-}3.19}$ Given $\mathbf{F} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ acting at $(1,1,1)$. Find the Torque about $(2,-1,5)$.

(a) Torque about a point is $\mathbf{r} \otimes \mathbf{F} = \mathbb{T}$ when $\mathbf{r}$ is from that point to point where $\mathbf{F}$ acts.



$$\mathbf{r}_1 = \mathbf{r}_2 + \mathbf{r}_3$$

so $\mathbf{r}_3 = \mathbf{r}_1 - \mathbf{r}_2$

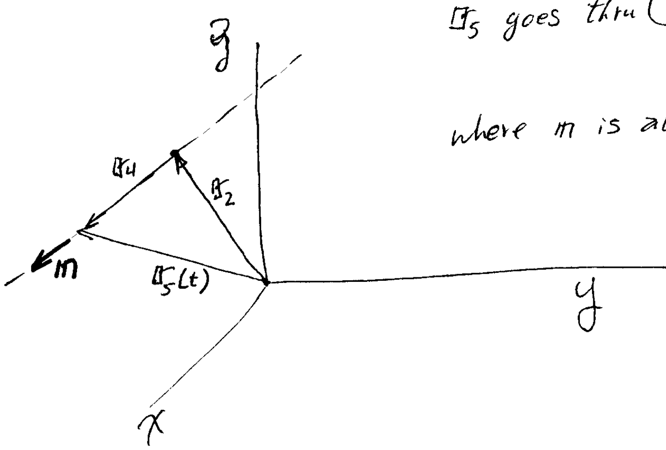
$$|\mathbf{r}_3| = (1,1,1) - (2,-1,5) = (-1,2,-4)$$

$$\mathbf{r}_3 \otimes \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 2 & -4 \\ 1 & 3 & 2 \end{vmatrix} = \boxed{16\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}}$$

(b) Find $\mathbb{T}$ about line $\mathbf{r}_5 = \overbrace{2\mathbf{i} - \mathbf{j} + 5\mathbf{k}}^{\mathbf{r}_2} + \overbrace{(\mathbf{i} - \mathbf{j} + 2\mathbf{k})}^{\mathbf{r}_4}t = \mathbf{r}_5(t)$

This line goes thru $(2,-1,5)$. All vectors $\mathbf{r}_5(t)$ end on this line which is along $\mathbf{r}_4$, not along $\mathbf{r}_5$

As the parameter t varies, the length of $\mathbf{r}_4$ varies, but not the direction. When $t=0$, $\mathbf{r}_4 = 0$ and $\mathbf{r}_5 = \mathbf{r}_2$, so indeed, $\mathbf{r}_5$ goes thru $(2,-1,5)$. The torque about $\mathbf{r}_5$ is



$$\mathbf{m} \cdot (\mathbf{r} \otimes \mathbf{F})$$

where $\mathbf{m}$ is along $\mathbf{r}_4$. $\quad \mathbf{m}_4 = \frac{\mathbf{r}_4}{r_4} = \frac{1}{\sqrt{6}}(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$

$\mathbf{m}_4 \cdot \mathbf{r}_3 \otimes \mathbf{F} = \mathbf{m}_4 \cdot (16\mathbf{i} - 2\mathbf{j} - 5\mathbf{k})$ since $\mathbf{r}_3$ is from point along line of interest

$$= \frac{1}{\sqrt{6}}(\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \cdot (16\mathbf{i} - 2\mathbf{j} - 5\mathbf{k})$$

$$\boxed{\mathbf{m}_4 \cdot \mathbf{r} \otimes \mathbf{F} = 8/\sqrt{6}}$$