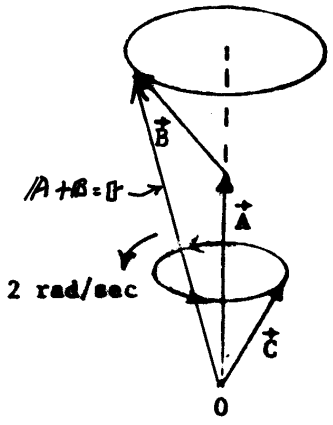
Chap 6
Section 3

6-3.6

#6. The diagram is just schematic, that is, the given $\vec{A}$, $\vec{B}$, $\vec{C}$ are not used in drawing the diagram. Think of the whole figure as rotating about $\vec{A}$ at 2 rad/sec. Then the velocity of the head of $\vec{B}$ is $\vec{v} = \vec{\omega} \times \vec{r}$, where $\vec{\omega} = \frac{2\vec{A}}{|\vec{A}|}$ (that is, a vector of length 2 in direction $\vec{A}$) and $\vec{r} = \vec{A} + \vec{B}$ (that is, a vector from 0 to the head of $\vec{B}$). Then



$$\vec{v} = \frac{2}{|\vec{A}|}\vec{A} \times (\vec{A} + \vec{B}) = \frac{2}{|\vec{A}|}(\vec{A} \times \vec{B}) \qquad \text{since } \vec{A} \times \vec{A} = 0.$$

Using the given vectors $\vec{A}$ and $\vec{B}$, we find $|\vec{A}| = \sqrt{1+1+4}$, and

$$\vec{v} = \frac{2}{\sqrt{6}}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -2 \\ 2 & -1 & 3 \end{vmatrix} = \frac{2}{\sqrt{6}}(\vec{i} - 7\vec{j} - 3\vec{k}).$$

The torque of $\vec{B}$ about the head of $\vec{C}$ is $\vec{r} \times \vec{F}$ where $\vec{F} = \vec{B}$ and $\vec{r} = \vec{A} - \vec{C}$, that is, the vector from the head of $\vec{C}$ (point the torque is about) to the head of $\vec{A}$ (point of application of the force). Thus the vector torque $= (\vec{A} - \vec{C}) \times \vec{B}$.

The scalar torque about line $\vec{C}$ is $\vec{n} \cdot (\vec{r} \times \vec{F})$ where $\vec{n} = \frac{\vec{C}}{|\vec{C}|}$; thus for the scalar torque we find

$$\frac{\vec{C}}{|\vec{C}|} \cdot (\vec{A} - \vec{C}) \times \vec{B} = \frac{1}{\sqrt{26}}\begin{vmatrix} 0 & 1 & -5 \\ 1 & 0 & 3 \\ 2 & -1 & 3 \end{vmatrix} = \frac{8}{\sqrt{26}}$$