Boas 6-6.3

Find deriv. of $xy^2 + yz$ at $(1,1,2)$ in direction $2i - j + 2k$

$$n = \frac{1}{3}(2i - j + 2k)$$

$$\nabla\phi = i y^2 + j(z + 2xy) + k(y) \quad = \hat{i} + 4\hat{j} + \hat{k}$$

$$\nabla\phi \cdot n = \frac{2}{3}y^2 - \frac{1}{3}(2xy + z) + \frac{1}{3}2y$$

$$= \frac{2}{3}(1)^2 - \frac{1}{3}(2\cdot1\cdot1 + 2) + \frac{1}{3}2(1)$$

$$\nabla\phi \cdot n = \frac{2}{3} - \frac{4}{3} + \frac{2}{3} = 0$$

Boas 6-6.4

Find derivative of $ze^x \cos y$ at $(1, 0, \pi/3)$ in direction of the vector $i + 2j$ (x y z)

unit vector along A is $n = \frac{A}{|A|} = \frac{i + 2j}{\sqrt{5}}$

$$\nabla w = i\frac{\partial w}{\partial x} + j\frac{\partial w}{\partial y} + k\frac{\partial w}{\partial z}$$

$$= i(ze^x \cos y) + j(ze^x)(-\underbrace{\sin y}_{=0 \text{ at pt.}}) + k(e^x \cos y)$$

$$= \frac{\pi e}{3} i + e k$$

$$\nabla w \cdot n = \left[z e^x \cos y + 2 z e^x(-\sin y)\right]\frac{1}{\sqrt{5}}$$

at point:

$$\nabla w \cdot n = \left[\frac{\pi}{3} e^1 \cos(0) + 2\left(\frac{\pi}{3}\right) e^0 (\underbrace{-\sin 0}_{=0})\right]\frac{1}{\sqrt{5}}$$

$$\nabla w \cdot n = \frac{\pi e}{3}\frac{1}{\sqrt{5}} = \frac{\pi e}{3\sqrt{5}} \simeq 2.5$$