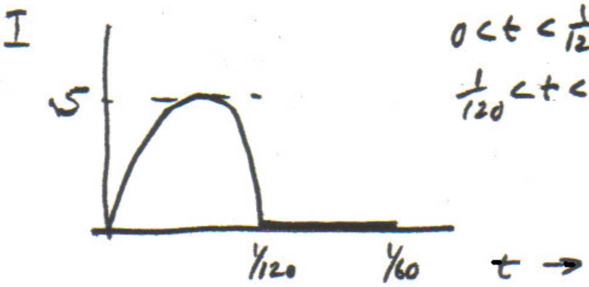
7-10.5



$0<t<\frac{1}{120}$ $\quad f(t)=5\sin(120\pi t)$

$\frac{1}{120}<t<\frac{1}{60}$ $\quad f(t)=0$

$2\ell=\frac{1}{60} \quad \ell=\frac{1}{120}$

$$a_n=\frac{1}{\ell}\int_0^{2\ell} f(t)\cos\frac{n\pi t}{\ell}\,dt \qquad f(x)=\frac{a_0}{2}+a_1\cos\frac{\pi x}{\ell}+\text{etc.}$$

for $n=0$

$$a_0=\frac{1}{1/120}\int_0^{1/120} 5\sin(120\pi/t)\,dt = 600\left[\frac{-\cos(120\pi t)}{120\pi}\right]_0^{1/120}$$

$$=\frac{-600}{120\pi}\left[(-1)-(1)\right]=\frac{5\cdot 2}{\pi}=\frac{10}{\pi}$$

$n=1$

$$a_n=600\int_0^{\frac{1}{120}}\sin(120\pi t)\cos(120\pi t)\,dt$$

$$=600\left[\frac{\sin^2(120\pi t)}{240\pi}\right]_0^{1/120}=\frac{600}{240\pi}\left[0\right]=0$$

$n\neq 1$

$$a_n=600\left\{-\frac{\cos[120\pi t(1-n)]}{240\pi(1-n)}-\frac{\cos[120\pi t(1+n)]}{240\pi(1+n)}\right\}_0^{1/120}$$

$$a_n=\frac{5}{2\pi}\left[\frac{-\cos[\pi(1-n)]}{(1-n)}-\frac{\cos[\pi(1+n)]}{1+n}+\underbrace{\frac{1}{(1-n)}+\frac{1}{1+n}}\right]$$

For n even;

$$a_n=\frac{5}{2\pi}\left[\frac{-(-1)}{1-n}-\frac{(-1)}{1+n}+\frac{1}{1-n}+\frac{1}{1+n}\right]$$

$$a_n=\frac{5}{2\pi}\left[\frac{1}{1+n}+\frac{1}{1-n}+\frac{1}{1-n}+\frac{1}{1+n}\right]$$

$$a_n=\frac{5}{2\pi}\left[\frac{2}{1+n}+\frac{2}{1-n}\right]=\frac{5}{\pi}\left[\frac{(1+n)+(1-n)}{(1+n)(1-n)}\right]$$

$$\boxed{a_n=\frac{5}{\pi}\left[\frac{2}{1-n^2}\right] \quad n \text{ even}}$$

For,
n odd:

$$\left[-\frac{\cos(-2\pi)}{1-n}-\frac{\cos(2\pi)}{1+n}+\frac{1}{1-n}+\frac{1}{1+n}\right]$$

$$\left[\left(\frac{-1}{1-n}\right)-\left(\frac{1}{1+n}\right)+\left(\frac{1}{1-n}\right)+\left(\frac{1}{1+n}\right)\right]=0$$

$$\boxed{\therefore\ a_n=0 \quad n \text{ odd}}$$

7-105 **Page 2**

$$b_n = 600 \int_0^{\frac{1}{120}} \sin(120\pi t) \sin(120 n\pi t)\, dt$$

$n = 1$ $$b_1 = 600 \int_0^{1/120} \sin^2(120\pi t)\, dt$$

$$= 600 \left[\frac{t}{2} - \frac{1}{480\pi} \sin(240\pi t) \right]_0^{1/120}$$

$$b_1 = 600 \left[\frac{1}{240} \right] = \frac{5}{2}$$

$n \neq 1$ $$b_n = 600 \left[\frac{\sin 120\pi t(1-n)}{240\pi(1-n)} - \frac{\sin 120\pi t(1+n)}{240\pi(1+n)} \right]_0^{1/120}$$

$$b_n = 600 \left[\frac{\overset{=0}{\sin(\pi(1-n))}}{240\pi(1-n)} - \frac{\sin(0)}{240\pi(1+n)} \right]$$

$b_n = 0$ since $\sin k\pi = 0$ where k is an integer $= 1-n$

Hence

$$I(t) = \overset{a_0/2}{\frac{5}{\pi}} + \frac{5}{\pi} \left[\sum_{n=\text{even}} \overbrace{\left(\frac{1}{n+1} - \frac{1}{1-n} \right)}^{a_n} \cos(120 n\pi t) \right] + \left[\overset{b_1}{\frac{5}{2}} \sin 120\pi t \right]$$

$$I(t) = \frac{5}{\pi} \left[1 + \sum_{\substack{n \\ \text{even}}} 2(1-n^2)^{-1} \cos(120 n\pi t) \right] + \frac{5}{2} \sin 120\pi t$$