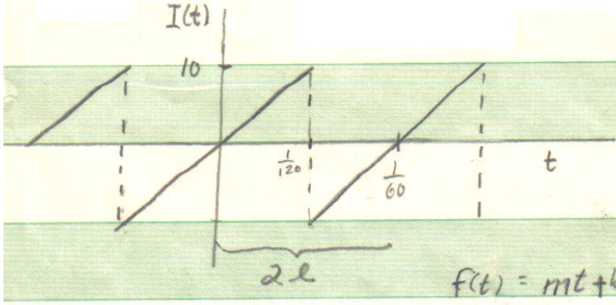
7-10.7



Period $= \frac{1}{60}$

$2\ell = \frac{1}{60}$

$\ell = \frac{1}{120}$

$f(x)$ is odd, so $a_n = 0$

$f(t) = mt + b \qquad m = \frac{10}{1/120} = 1200 \qquad b = 0$

$$b_n = \frac{2}{\ell}\int_0^{\ell} f(x) \sin \frac{n\pi x}{\ell}\, dx \Rightarrow \frac{2}{1/120}\int_0^{1/120} 1200t \sin \frac{n\pi t}{1/120}\, dt$$

$$b_n = (240)(1200)\int_0^{1/120} t \sin(n\pi \cdot 120 \cdot t)\, dt$$

let $x = 120 n\pi t \qquad dx = 120 n\pi\, dt$

then:

$$b_n = (240)(1200)\int_0^{1/120} \frac{x}{120 n\pi} \sin x \frac{dx}{120 n\pi}$$

$$b_n = \frac{20}{n^2\pi^2}\Big[\sin x - x\cos x\Big]_0^{1/120} = \frac{20}{n^2\pi^2}\Big[\overset{=0}{\sin n\pi} - n\pi \overset{=\pm 1}{\cos n\pi} - \overset{=0}{\sin 0} + 0\Big]$$

$$b_n = \frac{20}{n^2\pi^2}\left(-n\pi \cos n\pi\right) = \frac{-20}{n\pi}\left[\cos n\pi \begin{cases} +1, & \text{for } n \text{ even} \\ -1, & \text{for } n \text{ odd}\end{cases}\right]$$

$$b_n = \frac{-20}{n\pi}(-1)^n$$

Hence $\qquad f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{\ell}$

$$= \frac{-20}{\pi}\sum_{n=1}^{\infty} (-1)^n \frac{\sin 120 n\pi t}{n}$$