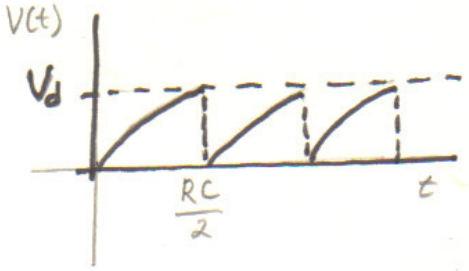
Boas7-13.4a



$$f(t) = g(t) = cV\left(1 - e^{-t/RC}\right) \simeq cV \sum_{-\infty}^{\infty} c_n e^{in\pi t/\ell}$$

Period $= 2\ell = RC/2 \Rightarrow \ell = \frac{RC}{4}$

$$c_n = \frac{1}{2\ell}\int_0^{2\ell} f(t) e^{-in\pi t/\ell}\,dt = \frac{1}{2\ell}\int_0^{2\ell}\left(1 - e^{-t/RC}\right)e^{-in\pi t/\ell}\,dt$$

$$c_n = \frac{2}{RC}\int_0^{RC/2}\left(1 - e^{-t/RC}\right)e^{-4\pi nit/RC}\,dt$$

for $n=0$

$$c_0 = \frac{2}{RC}\int_0^{RC/2}\left(1 - e^{-t/RC}\right)dt = \frac{2}{RC}\left[\int_0^{RC/2} dt - \int_0^{RC/2} e^{-t/RC}\,dt\right]$$

$$= \frac{2}{RC}\left[\left(\frac{RC}{2}\right) + RC\left(e^{-1/2} - 1\right)\right] = 1 + 2\left(e^{-\frac{1}{2}} - 1\right)$$

$$c_0 = 2e^{-\frac{1}{2}} - 1$$

for $n \neq 0$

$$c_n = \frac{2}{RC}\left[\underbrace{\int_0^{RC/2} e^{-4\pi int/RC}\,dt}_{c_n'} - \underbrace{\int_0^{RC/2} e^{-t/RC} e^{-4\pi int/RC}\,dt}_{c_n''}\right]$$

$$c_n' = \left(\frac{2}{RC}\right)\left(\frac{RC}{-4\pi in}\right)e^{-4\pi nit/RC}\Big|_0^{RC/2} = \frac{1}{-2\pi ni}\left(e^{-2\pi ni} - e^0\right) = 0$$

($e^{-2\pi ni} = 1$ since n = integer)

$\therefore\ c_n = \frac{-2}{RC} c_n''$

$$c_n = -\left(\frac{2}{RC}\right)\int_0^{RC/2} e^{-\frac{t}{RC}(1+4\pi ni)}\,dt = -\left(\frac{2}{RC}\right)\left(\frac{-RC}{1+4\pi ni}\right)\exp\left(-\frac{t}{RC}\right)(1+4\pi ni)\Big|_0^{RC/2}$$

$$c_n = \frac{2}{1+4\pi ni}\left[e^{-\frac{RC}{2RC}(1+4\pi ni)} - e^0\right] = \frac{2}{1+4\pi ni}\left[e^{-1/2}\underbrace{e^{-i2n\pi}}_{1 \text{ for all } n} - 1\right]$$

$$c_n = \left(\frac{2}{1+4\pi ni}\right)\left(e^{-1/2} - 1\right)$$

Boas 7-13.4a page 2

Hence

$$f(x) = CV \sum_{-\infty}^{\infty} \underbrace{\left(\frac{2}{1+4\pi n i}\right)\left(e^{-1/2}-1\right)}_{C_n} e^{4\pi i n t/RC} \quad \text{with } l = RC/4$$

$$= CV\left[\underbrace{1+2\left(e^{-1/2}-1\right)}_{C_0} + 2\left(e^{-1/2}-1\right) \sum_{\substack{-\infty \\ n\neq 0}}^{\infty} \frac{e^{4\pi i n t/RC}}{D_n}\right]$$

$$= CV\left\{1 + 2\left(e^{-1/2}-1\right)\left[1 + \sum_{\substack{-\infty \\ n\neq 0}}^{\infty} \frac{e^{4\pi i n t/RC}}{D_n}\right]\right\}$$

$$f(x) = CV\left[\left(\frac{2}{\sqrt{e}}-1\right)\left(1+\sum \frac{e^{4\pi i n t/RC}}{1+4\pi n i}\right)\right]$$