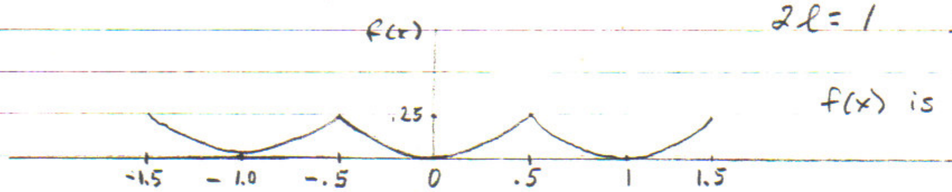
7 - 9.9

$f(x) = x^2 \quad -\frac{1}{2} < x < \frac{1}{2}$

$2\ell = 1 \quad \ell = 1/2$



$f(x)$ is even $\quad b_n = 0$

$$a_n = \frac{1 \cdot 2}{1/2} \int_0^{1/2} x^2 \cos n\pi x/{1/2}\, dx = 4 \int_0^{1/2} x^2 \cos 2n\pi x \, dx$$

$$a_n = \frac{4}{(2n\pi)^3} \Big[2(2n\pi x) \cos 2n\pi x + (4n^2\pi^2 x^2 - 2) \sin 2n\pi x \Big]_0^{1/2}$$

$$a_n = \frac{4}{(2n\pi)^3} \Big[2n\pi \cos n\pi + (n^2\pi^2 - 2) \overset{=0}{\sin n\pi} - 0 - (0-2) \overset{=0}{\sin 0} \Big]$$

$$a_n = \frac{4}{(2n\pi)^3} \Big[2n\pi (-1)^n \Big] = \frac{(-1)^n}{n^2 \pi^2}$$

$$a_0 = 4 \int_0^{1/2} x^2 \, dx = \frac{4}{3} x^3 \Big|_0^{1/2} = \frac{4}{3 \cdot 2^3} = \frac{4}{24} = \frac{1}{6} \quad \therefore \frac{1}{2} a_0 = \frac{1}{12}$$

$$f(x) = \frac{1}{2} a_0 - \frac{1}{\pi^2} \sum_{n=1}^{\infty} \frac{-(-1)^n}{n^2} \cos 2n\pi x$$

$$f(x) = \frac{1}{12} - \frac{1}{\pi^2} \Big[\frac{-(-1)^1}{1^2} \cos 2\pi x - \frac{(-1)^2}{2^2} \cos 4\pi x - \ldots \text{etc} \Big]$$