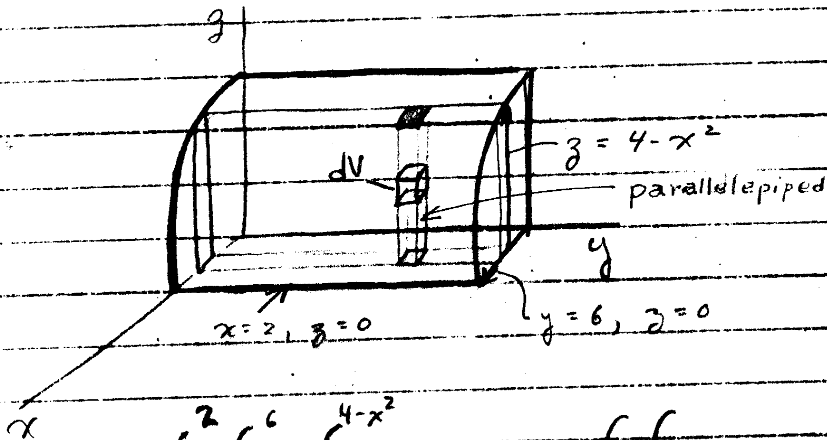
KJP-515

Find the (a) volume and (b) centroid of the region bounded by the parabolic cylinder $z = 4 - x^2$ and the planes $x=0$, $y=0$, $y=6$, $z=0$. Assume the density is 4.0 gm/cm³ everywhere.



a) $$V = \int_{x=0}^{2}\int_{y=0}^{6}\int_{z=0}^{4-x^2} dz\,dy\,dx = \int_x\int_y (4-x^2)\,dy\,dx = \int_x \Big[(4-x^2)y\Big]_{y=0}^{6} dx$$

Can do x/y integral 1st.

$$V = \int_{x=0}^{2} (24-6x^2)\,dx = 32$$

b) $$\bar{x} = \frac{1}{32}\iiint x\,dz\,dy\,dx = \frac{1}{32}\int\int \big[x(4-x^2)\big]dy\,dx = \frac{1}{32}\int\int_{y=0}^{6} (4x-x^3)\,dy\,dx$$

$$= \frac{6}{32}\left[\int_x 4x\,dx - \int_x x^3\,dx\right] + \frac{6}{32}\left[\frac{4x^2}{2}\Big|_0^2 - \frac{x^4}{4}\Big|_0^2\right] = 3/4 = 0.75$$

$$\bar{y} = \frac{1}{32}\iiint y\,dz\,dy\,dx = \frac{1}{32}\int_x (4-x^2)\left[\frac{y^2}{2}\right]_0^6 dx = \frac{18}{32}\int_0^2 (4-x^2)\,dx = 3.0$$

$$\bar{z} = \frac{1}{32}\iiint z\,dz\,dy\,dx = \frac{1}{32}\int_0^6 dy\int_0^2 \left(\frac{16-8x^2+x^4}{2}\right)dx = \frac{8}{5} = 1.6$$