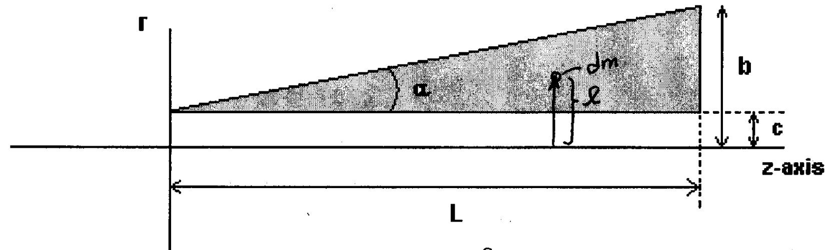
RJP-545

Find the moment of inertia about the z axis of the solid of revolution formed by a right triangle offset from the z-axis by distance c as depicted below. The length of the triangle is L.

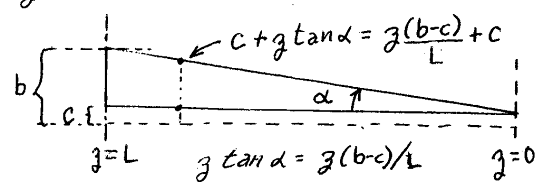


$$I_z = \int dI_z = \int \ell^2 dm = \rho \int r^2 dV$$ Use cylindrical coords., $dV = r\,dr\,d\theta\,dz$

$$I_z = \rho \int_r \int_\theta \int_z r^3 dr\, d\theta\, dz$$ Assuming constant ρ. Also, α is a constant.

$$I_z = \rho \int_{\theta=0}^{2\pi} \int_{z=0}^{L} \int_{r=c}^{c+z\tan\alpha} r^3 \, dr\, dz\, d\theta$$

$$I_z = 2\pi\rho \int_z \int_r r^3 dr\, dz$$



$$I_z = 2\pi\rho \int_z \left[\frac{r^4}{4}\right]_c^{c+z\tan\alpha} dz = 2\pi\rho \int_z \left[(c+z\tan\alpha)^4 - c^4\right] dz$$

Expand $(c+z\tan\alpha)^4$ and integrate term by term:

$$I_z = 2\pi\rho \int_{z=0}^{L} \left[(4c^3\tan\alpha)z + (6c^2\tan^2\alpha)z^2 + (4c\tan^3\alpha)z^3 + (\tan^4\alpha)z^4\right] dz$$

$$I_z = \frac{2\pi\rho}{4} L^2 \tan\alpha \left[2c^3 + 2c^2 L \tan\alpha + \frac{4cL^2}{4}\tan^2\alpha + \frac{L^3}{5}\tan^3\alpha\right]$$

Substitute for $\tan\alpha = (b-c)/L$

$$I_z = \frac{2\pi\rho}{4} L (b-c)\left[2c^3 + 2c^2(b-c) + c(b-c)^2 + \frac{1}{5}(b-c)^3\right]$$

Multiplying out the terms and simplifying, we get:

$$I_z = \frac{\pi\rho L (b-c)}{10} \underbrace{\left[b^3 + 2b^2c + 4c^3 + 3bc^2\right]}_{\left[(b+c)^3 + 3c^3 - b^2c\right]}$$

or, after expanding and simplifying:

$$I_z = \frac{\pi\rho L}{10} \left[2b^4 + 4cb^2 + 2c^3 b + 2b^2c^2 - 2cb^3 - 3c^4\right]$$