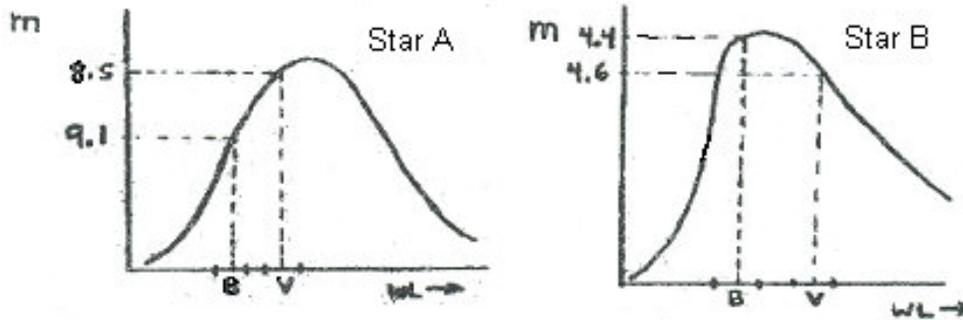


### 1-1 H. Color Index:

A **color index** is the difference of two color magnitudes, e.g., B-V, U-B, etc. A color index that is used very often is the B-V index. The diagram below illustrates the spectral distributions for two stars



with different temperatures. The B-V color index for star A is  $9.1 - 8.5 = +0.6$ , whereas the B-V color index for star B is  $4.4 - 4.6 = -0.2$ . Now star B is the hotter star with a surface temperature of 20,000 K and star A is cooler with a surface temperature of 5500 K. Notice that the hotter star has negative color index while the cooler star has a positive color index. That is, hot stars tend to be brighter in the blue part of the spectrum than in the visual or red part of the spectrum. For such stars,  $B < V$ . Cooler stars are brighter in the visual bandpass than in the blue bandpass so  $V < B$ . Hence, color indices convey useful information about a star's spectrum and temperature. Values of B-V have been calibrated to directly indicate the temperature of a star. An abbreviated table is given below.

B-V	Temp (K)
-0.30	33,000
-0.15	14,800
0.00	9,600
+0.50	6,500
+1.00	4,800
+1.52	3,600

### 1-1 I. Absolute or Intrinsic Brightness (Luminosity and Absolute Magnitudes):

This is the true brightness of an object, independent of its distance. The intrinsic brightness of a star depends only on the following physical parameters:

1. Surface temperature.
2. Radius or surface area

**Luminosity** is an expression of intrinsic brightness. It is the total radiative power output of a star and may be given in Watts or ergs per second, just like a radio station or light bulb.

$$L_* = 4\pi R^2 \sigma T^4 \quad (1-17)$$

Here R is the radius of the star and T is the surface temperature. The factor  $\sigma T^4$  is the integrated or bolometric surface brightness or flux of a star and is known as the **Stefan-Boltzmann Law**. This will

be discussed further in the next chapter. The luminosities of stars are usually given in terms of the Sun's luminosity, e.g.  $L_* = 8.00L_{\odot}$ . Do **RJP-8**.

**Absolute magnitude** is also an expression of intrinsic brightness. It is the magnitude of an object when seen from a distance of 10 parsecs (See Appendix A on measuring stellar distances). However, the absolute magnitude scale is a relative scale of absolute or intrinsic brightness. Astronomers use absolute magnitudes to express which stars are truly bright and which are truly faint, because distance is no longer a variable.

The absolute magnitude scale works the same way the apparent magnitude scale works. For example, a star that is 100 times more luminous than the Sun would have an absolute magnitude that is 5 magnitudes brighter than the Sun's absolute magnitude (remember, a step of 5 mags. is defined to correspond to a brightness ratio of exactly 100). Since the Sun's absolute magnitude is approximately +5 (4.79 to be exact), a star with a luminosity 100 times the Sun's would have a value of  $M_* = 0$ .

Absolute magnitude,  $M$ , is a number that can only be computed, not measured. To compute  $M$  for a star we must:

1. Measure the apparent magnitude of the star.
2. Determine the distance of the star.

The distance of a star may be calculated using trigonometry, if a very small angle called the parallax of the star can be measured. (The parallax is the annual, semi angular displacement of the star in the sky due to the Earth's revolution. See Appendix A and Motz, Chap. 1, page 3-5). Knowing the distance and apparent magnitude of a star, one can use the inverse square law to compute what its magnitude would be at 10 pc using the inverse square law for the diminution of brightness. Let  $B_M$  be the brightness of an object when observed from a distance of 10pc and  $b_m$  the brightness of the same object when observed from a distance  $d$ . Then the inverse square law gives:

$$\frac{b_m}{B_M} = \left( \frac{10}{d} \right)^2 \quad (1-18)$$

But according to the definition of the magnitude scale,

$$b_m/b_M = 2.512^{(M-m)} = [10^{0.4}]^{(M-m)}$$

So,

$$(10/d)^2 = [10^{0.4}]^{(M-m)}$$

Take the log of both sides:

$$2(\log 10 - \log d) = (M-m) (0.4) \log 10$$

$$(2-2\log d)/0.4 = M-m$$

$$5-5\log d = M - m$$

Solve for  $M$ :

$$\mathbf{M = m + 5 - 5\log (d)} \quad (1-19)$$

Here  $d$  must be expressed in parsecs. Or, since  $d=1/\pi$ ,

$$M = m + 5 + 5 \log (\pi), \quad (1-20)$$

where  $\pi$  is the parallax in arcseconds (Motz uses  $p$  instead of  $\pi$ ). However, this can be done only for about 2,000 stars, all of which are within 100 pc of the Sun. It has been found that the absolute magnitudes of stars range from -10 (the intrinsically brightest stars) down to +18. The Sun's absolute visual magnitude is +4.79, making it an average star when compared with the other stars.

Do RJP-9, 10, 11.

### 1-1 J. Bolometric Magnitudes

A bolometric magnitude,  $m_{\text{bol}}$  (or  $M_{\text{bol}}$ ), is the magnitude found by measuring the observed flux over the entire EM spectrum. Needless to say, this is very difficult to do. Bolometric magnitudes are always brighter than magnitudes measured for some bandpass (smaller in value). For example  $M_V$  for the Sun is +4.79, whereas its absolute bolometric magnitude is  $M_{\text{bol}} = +4.72$ .

As the radiation from a body travels outwards from the surface, it must pass through successive concentric spheres of larger and larger surface area. Using the conservation of energy for the luminosity of a star we have

$$L_* = 4\pi R^2 \sigma T^4 = 4\pi r^2 \int F_\lambda(r) d\lambda = 4\pi r^2 F_{\text{bol}}(r), \quad (1-21)$$

Where  $F_{\text{bol}}(r)$  is the total integrated (over all wavelengths) or bolometric flux arriving on a sphere at a distance  $r$  from the star. Therefore,

$$F_{\text{bol}}(r) = L_*/4\pi r^2. \quad (1-22)$$

### 1-1 K. Bolometric Corrections

The bolometric correction is defined as

$$BC = M_{\text{bol}} - M_V \quad (1-23)$$

For example, the bolometric correction for the Sun is  $4.72 - 4.79 = -0.07$ . Bolometric corrections are always negative and are usually computed from theory. They may be found tabulated in various sources, even on line.

Do RJP-12.

### 1-1L. Distance Modulus:

Distance modulus is an indicator of the distance of an object expressed in terms of its absolute and apparent magnitudes. More specifically, distance modulus is defined as  $m-M$ . If the distance modulus is 0, the object is 10 parsecs distant. If  $m-M$  is less than zero or negative, this means the object is closer than 10 parsecs. If  $m-M$  is positive, then the object is farther than 10 parsecs. Distance modulus is actually a logarithmic index of distance. Do RJP-13.

## 1-1M. Selective Absorption and Reddening

In addition to distance affecting the apparent brightness of an object, further dimming is caused by absorption within the interstellar medium. In general, the more distant the object, the greater the amount of absorption, but it also depends on the line of sight to the object through the galaxy. The absorption is also wavelength dependent, with the result that an object is reddened, just as the Sun and Moon are when seen towards the horizon. The result is that the measured color index,  $(B-V)_m$ , is increased relative to its intrinsic value,  $(B-V)_o$  or  $(B-V)_i$ . Intrinsic values may be computed from the black body theory of radiation and may be found tabulated for a given temperature. Let  $\Delta m_V$  be the diminution of the star in magnitudes and  $E_{BV}$ , **the color excess**, defined as

$$E_{BV} = (B-V)_m - (B-V)_o \quad (1-24)$$

A general rule that is sometimes used is

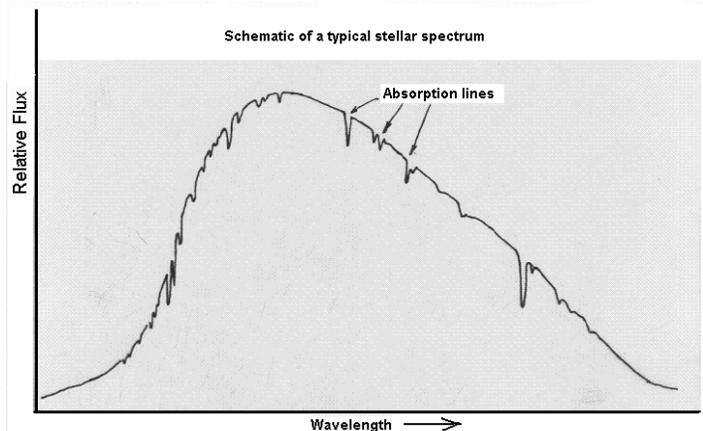
$$\Delta m_V = 3E_{BV} \quad (1-25)$$

But this is not very dependable, except for doing PHY466 problems. Do **RJP-14, 15**.

## 1-2. Stellar Spectral Classification

To a first approximation, stars radiate as black bodies, as given by Planck's Law for an incandescent gas under high pressure. Accordingly, the spectrum should show a smooth and continuous variation of flux with wavelength, that is, a black body spectrum. However, this is not the case. The schematic diagram to the right depicts a typical stellar spectrum. There one sees that stellar spectra exhibit numerous absorption lines superposed on a continuum that may be approximated by a black body spectrum for a specific temperature. The continuum is radiated by what is called the

**photosphere** or surface of the star, which is considered to be an incandescent gas under high pressure. The absorption lines are produced when the photospheric radiation passes through the star's lower atmosphere, or **chromosphere**, which is cooler than the photosphere.



In 1920, the director of the Harvard College Observatory, E. C. Pickering, and his assistants, most notably Annie. J. Cannon, began a project of photographing and classifying the spectra of thousands of stars. The result was a catalog of nearly 200,000 stellar spectra called the Henry Draper Catalog, named after a wealthy physician, who bequeathed the funds for the project.

The original classification scheme was based on the strengths of the Balmer absorption lines for hydrogen and letters ranging from A to P were assigned accordingly. However, with the advent of theoretical work in statistical quantum physics by Saha in 1926, it was realized that line strength depends primarily on the temperature of the star's atmosphere. It was then decided to make adjustments in the classification scheme so that it was a temperature sequence. Some spectral classes were then eliminated and the surviving letters subdivide into decades. The most common spectral classes are **O, B, A, F, G, K, and M**. This is in order of decreasing surface temperature. Each of these

classes is further subdivided into decades, such as B2, A1, K5, or M8. A B0 star is slightly hotter than a B1 star and an A5 star is somewhat hotter than an A6 star, etc. There are also spectral types O9.5, which is hotter than B0, and B0.5, which is hotter than B1. The Sun's spectral class is G2. The diagram below displays the appearance of stellar spectral over the wavelength range from 350nm to 750nm for a wide range of spectral classes.

Spectral class is often used as a surrogate for temperature, as is **color index**.

It is also known from laboratory studies, exactly what wavelengths are absorbed or emitted by every atom. Hence, by examining which absorption lines are present in the spectrum of a star, it possible to determine the chemical composition of the star's atmosphere. This is called spectrochemical analysis. The principle of spectrochemical analysis is "Every atom emits or absorbs radiation at a unique set of wavelengths, depending on its atomic structure."

### 1-3. Luminosity Classes

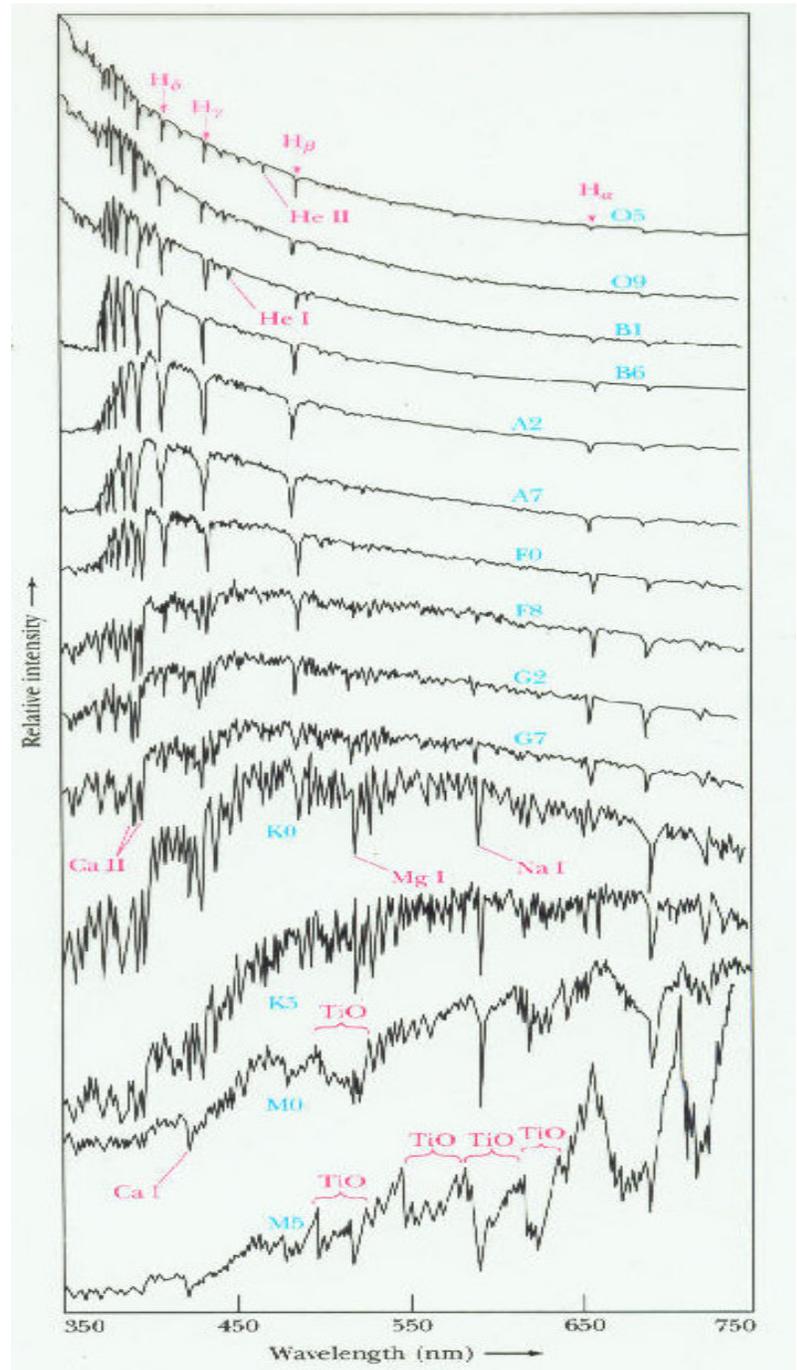
Stars may be classified according to their luminosity, which is, to some degree, related to evolutionary stage. The luminosity class may be determined by examining the widths of the absorption lines in a star's spectrum. This matter will be taken up later. The usual luminosity classes are:

- Ia, b, & c. Supergiants
- II. Bright Giants
- III. Giants
- IV. Sub-giants
- V. Main Sequence
- VI. Sub-dwarfs
- VII. White Dwarfs

Intrinsic color indices are different for the different luminosity classes for a given temperature. See Appendix B. Do **RJP-16, 17, 18, 19, 20.**

### 1-4. The H-R Diagram

Between 1908 and 1913, E. Hertzsprung, a Danish engineer and astronomer, and H. N. Russell in the US, independently compiled absolute magnitudes and color indices for many stars and displayed this information graphically. Such a graph, where M is plotted versus color index or spectral class, is now called a Hertzsprung-Russell or H-R diagram. An example is displayed below. Here temperatures



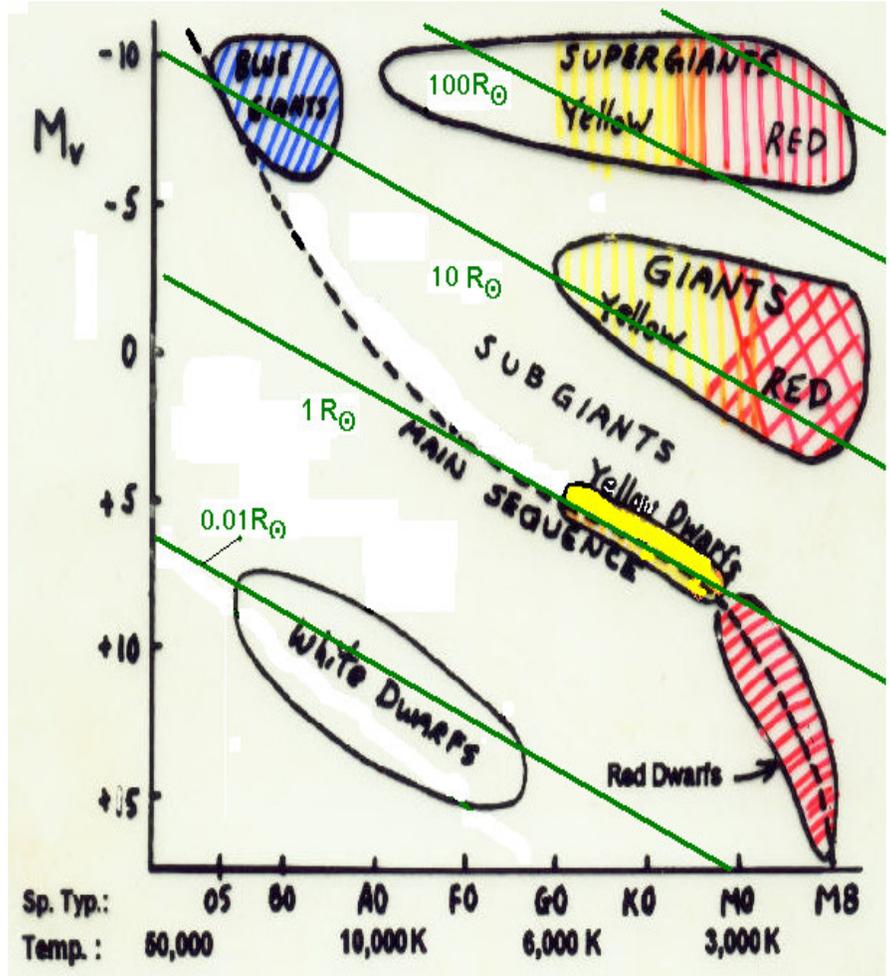
corresponding to the spectral classes are also shown, but not color indices. Loci of constant radius are also shown.

To plot a star as point in this diagram, one must measure the parallax to convert the apparent magnitude to absolute magnitude. This can be done for about 2,000 stars.

One can see that stars do not occupy places in the H-R diagram randomly. Instead, the properties of stars are such that they tend to populate the diagram in various groupings, such as the white dwarfs, main sequence stars, red giants, blue giants, etc. It is now realized that these groupings represent relatively long lived stages of stellar evolution. That is, statistically one expects to find more stars in a long lived stage of evolution than in a short lived one. The main sequence stage is a relatively long lived stage of evolution, as is the white dwarf stage.

Stars on the main sequence are delineated by mass. The greater the mass of a main sequence star, the brighter it is. This is called the **Mass-Luminosity Law**. This law does not apply to any other grouping or stage of evolution.

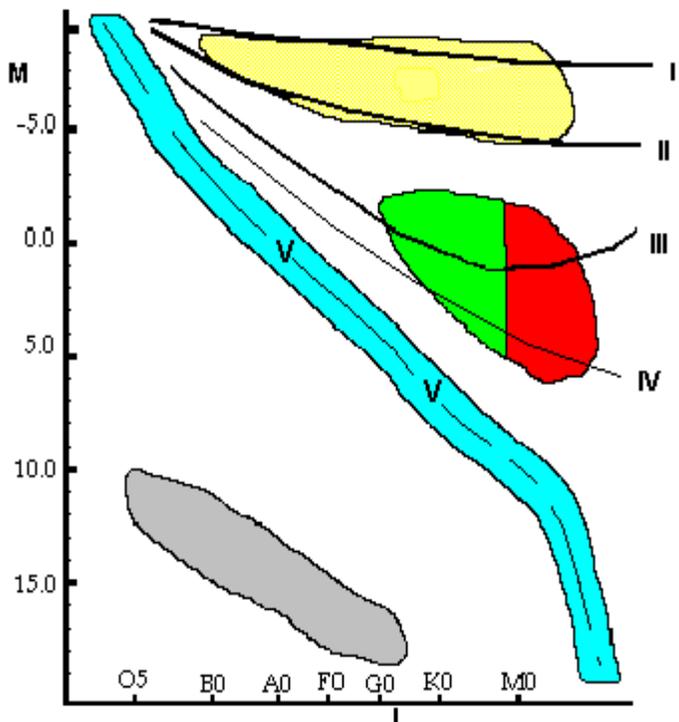
When we add the loci for the luminosity classes to an H-R Diagram, we get the version of the H-R Diagram shown below.



### 1-5. Spectroscopic Parallax

By determining both the spectral type and luminosity class, a star's location in the H-R Diagram may be determined without knowing the absolute magnitude *a priori*. For example, a K2 III star could be located in the H-R diagram as a point on the locus labeled III, and hence, that star's absolute magnitude may be read from M-scale. This procedure may be employed to determine the distance of a star whose parallax is too small to be measured, and is called the spectroscopic parallax method.

Using the H-R diagram to the right, or using a table, we may determine that the



absolute magnitude of a K2 III star is -0.05. If the apparent magnitude is measured, one may calculate the distance modulus of the star or the actual distance from equation (1-19).

The reality of the situation is that stars classified by luminosity class do not define a unique locus in the H-R diagram, but instead there is what is called cosmic dispersion. This is one of the largest sources of uncertainty in this method. One must also account for reddening, as described in section 1-1M.

Do **RJP-22**.