

CHAPTER 2

Part A

THEORY OF THERMAL RADIATION

2-1. Reflection Versus Emission

The terms light and electromagnetic radiation (EMR) are used interchangeably in what follows, though the term light is usually reserved for referring to EMR that is visible to the human eye. A clear distinction must be made between the processes of emission and reflection. We see things around us by reflection when they are cool ($<1000\text{K}$). In this case, the source of the light (the illuminating source) is external to the object we see. Stars, fires, and light bulbs are illuminating sources of white light, which is a mixture of all colors or wavelengths. The Moon, the planets, the walls of a room at room temperature, the entire terrestrial environment, and you are seen by reflected light.

If we see an object by reflection, it may absorb certain wavelengths and reflect certain others. Whenever radiation is absorbed, the absorbed radiation is converted into heat or thermal energy within the absorber. Since atoms are in constant motion, this energy is quickly shared by all the atoms making up the absorber. The greater the amount of kinetic energy there is per atom, the higher the temperature. This is the basis of the kinetic theory of matter. However, for a given T , the atoms are not all moving with the same speed. The latter depends on the mass of the particle.

2-2. Kinetic Theory

It was J. C. Maxwell and L. Boltzmann who developed the classical statistics of a system of particles, either atoms or molecules, in the latter part of the nineteenth century. Their work established the distribution of atomic speeds, and therefore the kinetic energy in a gas, for a given temperature.

2-2A. Maxwell-Boltzmann Velocity Distribution

Consider a confined gas in thermodynamic equilibrium (*TDE*) at temperature T . Assume the gas consists of atoms or molecules of mass m and that the molecules do not exert forces on one another except for a very brief moment during a collision. We also assume that the molecules undergo perfectly elastic collisions. Hence the atoms or molecules have only kinetic energy and no potential energy. Making such assumptions, both Maxwell and Boltzmann have shown that the probability that a particle has a speed v is

$$P(v) \propto e^{(-E/kT)}. \quad (2-1)$$

Here E is the kinetic energy, $E = \frac{1}{2}mv^2$, so:

$$P(v) \propto e^{(-mv^2/2kT)}. \quad (2-2)$$

The probability that an atom or molecule has a speed between v and $v+dv$ with a specific direction defined to be within a solid angle $d\omega$ that has orientation θ, ϕ with respect to a spherical coordinate system is

$$P(v, \theta, \phi) dv \propto (m/2\pi kt)^{3/2} e^{(-mv^2/2kT)} v^2 dv. \quad (2-3)$$

For the above equations, k is the Boltzmann constant and has a value 1.384×10^{-16} erg/Kelvin. The derivation of this expression is quite complex and will not be given here.

Now, it must be that the sum (integral) of all probabilities must be unity, that is:

$$\int_{v=0}^{v=\infty} \int_{\omega=0}^{\omega=4\pi} P(v, \theta, \varphi) dv d\omega = 1. \quad (2-4)$$

If space is isotropic, then all directions for the speeds are equally probable. This means that P is independent of θ and φ , and hence,

$$\int_0^{4\pi} P d\omega = P \int_0^{4\pi} d\omega = P(v)4\pi \quad (2-5)$$

This condition provides the required normalizing factor, 4π , so that (2-3) may be written as an equality. That is:

$$P(v)dv = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2kT} \right)^{3/2} e^{-E/kT} v^2 dv \quad (2-6)$$

The number of atoms per unit volume with speeds between v and $v+dv$ is then $dN(v)$ where

$$dN(v) = N_{\text{tot}} P(v) dv \quad (2-7)$$

$$\text{or} \quad \frac{dN(v)}{dv} = \frac{4N_{\text{tot}}}{\sqrt{\pi}} \left(\frac{m}{2kT} \right)^{3/2} v^2 e^{-mv^2/2kT} \quad (2-8)$$

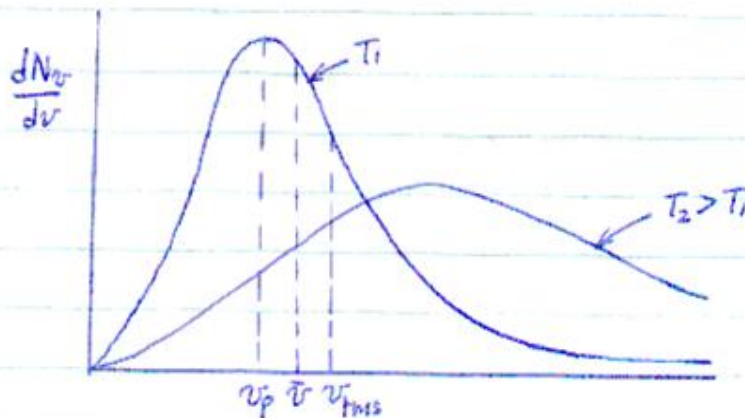


Fig. 1. The Maxwell-Boltzmann speed distribution function for two different temperatures.

The most probable speed, v_p , is found by differentiating (2-8), setting this to zero, and solving for v . One gets

$$\text{most probable: } v_p = \left(\frac{2kT}{m} \right)^{1/2} \quad (2-9)$$

$$\text{mean speed: } \bar{v} = \left(\frac{8kT}{\pi m} \right)^{1/2} \quad (2-10)$$

$$v_{\text{rms}} = \left(\frac{3kT}{m} \right)^{1/2} \quad (2-11)$$

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From the last expression, (2-11), we see that kinetic theory indicates that

$$\frac{1}{2} m v^2 = \frac{3}{2} k T \quad (2-12)$$

In this equation, v is the root mean square speed. Therefore, temperature is an index of the average kinetic energy per atom or molecule in a substance.

Temperature defined by (2-12) is referred to as **kinetic temperature**. The higher the temperature, the faster the atoms are moving in a gas or liquid, or the faster they are vibrating in place within a solid. The sum of all the kinetic energies of all the atoms in a body constitutes the heat or thermal energy in a body. So the amount of heat in a body depends on both the temperature and the mass of the body.

Do RJP-30.

2-2 B. Kinetic Theory and EM Radiation

In the mid nineteenth century, experimental evidence indicated that there was a direct correlation between temperature and the type and amount of radiation that a substance or body radiates. Furthermore, it was known that temperature was related to the apparent color of a radiating source of light: blue = hot, red = cooler. This is because very hot objects give off or emit much more blue light than red light, while cooler objects emit more red light than blue light. These relationships are summarized by what are called the laws of thermal or heat radiation, viz., Wien's Law, the Stefan-Boltzmann Law, and eventually Planck's law (1901). The basis of these laws is the fact that emitted light or electromagnetic radiation (EMR) comes from the conversion of thermal energy into radiant energy by atomic processes, more specifically, electron transitions. However, the latter was not known until 1915, when N. Bohr successfully developed the quantum model of the atom.

If the temperature of a body is sufficiently high ($>1000\text{K}$), thermal energy or heat is converted into a noticeable amount of visible radiation. Such bodies are then said to be incandescent. Incandescent light bulbs, stars, molten lava, a hot poker, and glowing embers in a fire are all incandescent bodies. Such objects radiate in accord with the laws of EMR and referred to as black bodies.

The term "black body" refers to the theory that at absolute zero a body would absorb all radiation falling on it and would emit no radiation. That is, it would appear black. All objects, regardless of their temperature, radiate some type and amount of electromagnetic radiation, even you. At low temperatures, the predominant radiation is infrared or weak microwaves. Things that emit visible light are the Sun, stars, light bulbs, and fires.

2-3. Laws of Radiation

Experimental data by Stefan and others during the mid-nineteenth century indicated that the spectral energy distribution of incandescent bodies was as shown in Fig. 1. An incandescent body was any solid, liquid or gas at high pressure heated to the point where it emitted a continuous spectrum that could be measured. Furthermore, the type and amount of electromagnetic radiation emitted per unit area per unit time (bolometric surface brightness) by an object

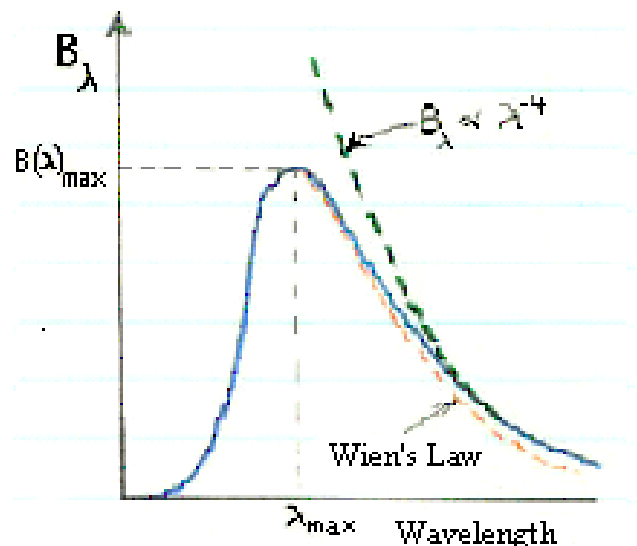


Fig. 2. Spectral distribution for an incandescent body

was found to depend only on that body's surface temperature. The higher the temperature, the greater the total amount of radiation emitted per unit area per second, or bolometric flux. This is given quantitatively by the Stefan-Boltzmann Law,

$$\text{Stefan-Boltzmann Law: } B_{\star} = \sigma T^4 \text{ (ergs/cm}^2\text{/sec),} \quad (2-13)$$

where σ is a known number or constant of proportionality between B and T . The asterisk subscript indicates that this is the bolometric surface brightness of a star. T must be the absolute temperature expressed in Kelvins. This relation was found empirically by Stefan and later derived theoretically by Boltzmann.

All theoretical attempts to derive a mathematical expression for the observed spectral distribution, as depicted in Fig. 1, based on theory failed. One was the Rayleigh-Jeans Law:

$$I_{\lambda} = B_{\lambda} \sim \lambda^{-4} = 2ckT/\lambda^4 \quad (2-14)$$

However, this produced what was called “the ultraviolet catastrophe,” namely, that the intensity of the radiation was predicted to go to infinity for shorter and shorter wavelengths, such as in the ultraviolet. However, the Rayleigh-Jeans Law is valid for:

$$\lambda > 0.3/T \text{ (cm)} \quad (2-15)$$

A somewhat more successful derivation by Wien yielded:

$$B_{\lambda} = (2hc^2/\lambda^5)e^{-hc/\lambda kT}, \quad (2-16)$$

where h is now known as Planck’s constant. This expression successfully yields which color or wavelength (λ_{\max}) is emitted with maximum intensity, viz., Wien's Displacement Law:

$$\lambda_{\max} = a/T, \quad (2-17)$$

where “ a ” is a constant of proportionality. Unfortunately, the Wien Law fails for $\lambda > \lambda_{\max}$. Success was finally achieved in 1900 by Max Planck, who introduced the idea that a system may have only discrete energy states rather than a continuum of possible energies.

Planck’s Theory

Imagine a cavity in which there is a vacuum and the walls of the cavity have an absolute temperature of T . Radiation is emitted by the walls of the cavity into the volume of the cavity and cannot escape. Rather, the radiation travels across the cavity and is continuously either reflected or absorbed and re-emitted by the walls. The radiation field in the cavity is in thermal equilibrium with the walls when the amount emitted is equal to the amount absorbed. The radiation field then is indicative of the temperature of the walls. The electrons in the atoms making up the walls of the cavity may be considered to be harmonic oscillators as a result of the radiation field. That is, the radiation field exerts forces on the electrons and causes them to be set in motion as a result of absorption and emission processes. The electrons then oscillate about some mean position within the atoms, which can be described by Hooke’s Law:

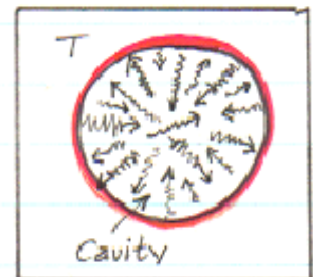
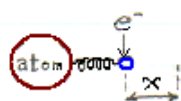


Fig. 3. A cavity containing a radiation field in thermal equilibrium with the walls.



$$m_e \frac{d^2 x}{dt^2} = -Kx$$

$$(2-18)$$

In this expression, K is Hooke's constant or the restoring force constant and x is the displacement of the electron from its equilibrium or static position. This is a 2nd order differential equation, the solution of which is

$$x = A \cos \omega t + B \sin \omega t = C e^{(-i\omega t)} \quad (2-19)$$

In this equation, $\omega = [K/m_e]^{1/2} = 2\pi\nu$ is the angular frequency of the oscillation.

From statistical thermodynamics, we have the Boltzmann distribution law, which gives the number of oscillators (atoms) in the n th energy level, ϵ_n , that are in thermal equilibrium with one another to be:

$$N_n = (N/Z)[e^{(-\epsilon_n/kT)}] \quad (2-20)$$

N is the total number of oscillators in the system and Z is the Boltzmann partition function. The later is found by the fact that the sum of N_n must be N . That is,

$$\sum_{n=1}^M N_n = N \quad (2-21)$$

M is the total number of energy levels. Hence,

$$\sum_n^M \frac{e^{-\frac{\epsilon_n}{kT}}}{Z} = 1,$$

or

$$Z = \sum_n^M e^{-\epsilon_n/kT} \quad (2-23)$$

The index n identifies what energy state is to be considered. At this point, n may range from 1 to $M=\infty$. Also, ϵ_n may have any value and is the sum of the kinetic and potential energies for a given state. However, T only indexes the KE and not the PE.

Now we find the average energy of N oscillators in the cavity at temperature T . This is:

$$\bar{\epsilon} = \sum_n \frac{N_n \epsilon_n}{N} \quad (2-24)$$

Now substitute for N_n from (2-20) to get

$$\bar{\epsilon} = \sum_n \frac{N \epsilon_n}{N Z} e^{-\epsilon_n/kT} = \sum_n \frac{\epsilon_n}{Z} e^{-\epsilon_n/kT} \quad (2-25)$$

Planck restricted the energy states to be quantized, such that,

$$\varepsilon_n = h\nu \left(n + \frac{1}{2} \right) \quad (2-26)$$

This was the beginning of quantum physics. Using (2-22) and (2-26) in (2-25) we obtain:

$$\bar{\varepsilon} = \frac{\sum_n h\nu \left(n + \frac{1}{2} \right) e^{-h\nu \left(n + \frac{1}{2} \right) / kT}}{\sum_n e^{-h\nu \left(n + \frac{1}{2} \right) / kT}} \quad (2-27)$$

Now the summation starts at $n=0$ instead of $n=1$. Examination of (2-27) reveals that the numerator is the derivative of the denominator with respect to $1/kT$ when we write (2-27) in the form:

$$\bar{\varepsilon} = - \frac{\partial}{\partial (1/kT)} \left[\ln \left(\sum_n e^{-h\nu \left(n + \frac{1}{2} \right) / kT} \right) \right] \quad (2-28)$$

$$\text{or } \frac{\partial}{\partial x} \left[\ln \sum e^{ax} \right] = \frac{\sum a e^{ax}}{\sum e^{ax}} \quad (2-29)$$

where $x = 1/kT$ and $a = h\nu \left(n + \frac{1}{2} \right)$. (Do RJP-31) The factor e^{ax} doesn't cancel in (2-29) because "a" depends on n . The argument of the \ln in (2-28) is of the form

$$\sum_n e^{-\varepsilon_n/kT} = \sum_n b x^n = b [1 + x + x^2 + x^3 + \dots] \quad (2-30)$$

where, $b = e^{-h\nu/(2kT)}$ and $x = e^{-h\nu/kT}$, using (2-26). Again, the summation begins with $n=0$. Now the quantity in the square brackets is just the binomial expansion of $(1-x)^{-1}$. Hence,

$$\sum_n e^{-\varepsilon_n/kT} = \frac{b}{(1-x)} \quad (2-31)$$

$$\text{or } \sum_n e^{-\varepsilon_n/kT} = e^{-h\nu/2kT} \left[\frac{1}{1 - e^{-h\nu/kT}} \right]$$

$$\text{Then } \ln \left(\sum_n e^{-\varepsilon_n/kT} \right) = - \frac{h\nu}{2kT} - \ln \left(1 - e^{-h\nu/kT} \right)$$

$$\text{since } \ln \left(\frac{1}{x} \right) = - \ln x \text{ where } x = 1 - e^{-h\nu/kT}$$