PHY 466 INTRODUCTORY ASTROPHYSICS
Homework Problems

See Appendix at end for values of constants.

RJP-0.5 Given that $I_\lambda = 2ckT/\lambda^4$ (the Rayleigh-Jeans Law) is isotropic, compute the radiant energy that emanates from a square cm per second from a surface at 1000K into the solid angle defined by $\theta = 40^o$ to 50^o, $\phi = 40^o$ to 50^o, for the wavelength interval from 400 nm to 700nm (the visible bandpass). Here c is the speed of light, k is Boltzmann's constant, and T is the temperature of the surface. Show setting up the double integral by hand, though you can look up the results in tables or use Mathematica. Work in cgs units.

RJP-0.6 A certain star has a radius of 3.15×10^8 cm. The measured flux for this star in the visible bandpass is 4.55×10^{-9} erg/cm^2/sec. Assume the surface intensity is given by the Rayleigh-Jeans Law, and the surface temperature is 8700K. Calculate the distance of the star in light years.

RJP-0.8 Write a FORTRAN program that numerically integrates the bandpass flux from 1620 to 1750 Angstroms for the data from the IUE spectral image SWP54407. The latter is an EXCELX data file to be found on this web site. Submit the source code for your program with the answer written somewhere at the end of the latter. There is no table of data to be submitted.

RJP-1.0 Compute how many times the apparent brightness of the Sun is greater than the apparent brightness of the star Sirius. Look up the apparent visual magnitudes of each object.

RJP-2.0 Find the difference in magnitudes for 2 stars that have a brightness ration of 4.00.

RJP-3.0 Star A has a visible band flux , $F_V = 4 \times 10^{-3}$ erg/cm^2/sec, while star B has $F_V = 1.8 \times 10^{-3}$ erg/cm^2/sec. If star A has $m_V = 8.32$, calculate the value of m_V for star B.

RJP-4.0 Find the apparent magnitude of the Sun when seen from a distance of 65,000 AU.

RJP-4.5 Determination of magnitudes from observational data. Download 2 documents.

RJP-5.0 The Alpha Centauri (α Cen) system has an observed visual magnitude equal to - 0.01 and contains 2 identical stars. What are the individual apparent visual magnitudes of the components?

RJP-6.0 Find the combined magnitude for a binary star system in which the separate magnitudes of the component stars are 3.00 and 4.00.

RJP-7.0 A very distant star cluster contains 10,000 member stars, 100 of which have M_v=10.0 and the remainder have M_v=15.00. Calculate the combined absolute magnitude of the cluster.

RJP-8.0 If the distance of the cluster in problem 7 is 5 kpc, what is the observed combined apparent visual magnitude of the cluster?

RJP-9.0 If the surface temperature of the Sun is 5800K and its radius is 6.98×10^5 km, calculate the luminosity of the Sun is watts.

RJP-10.0 If the measured parallax for α Cen is 0.72", what is the absolute magnitude of the system?

RJP-11.0 If you were an astronomer observing from a planet in the α Cen system, what would be the apparent visual magnitude of the Sun? Assume the distance of the planet from its 2 stars is negligible compared to the heliocentric distance of the system.

RJP-12.0 Derive an expression for the uncertainty in absolute magnitude, dM, for a given uncertainty in the parallax, $d\pi$, then compute dM for $d\pi = 0.001"$.

RJP-13.0 A. Calculate the distance modulus and absolute visual magnitude for a star that has an apparent visual magnitude of -0.40 and a parallax of 0.300 arcseconds.

B. Calculate the apparent visual magnitude of the star as seen at a distance of 1 kiloparsec.

RJP-14.0 A star is observed to have $m_V = 8.30$, $m_B = 8.00$, and a parallax of 0.002." The BC for this star is known to be -0.55 and $(B-V)o = -0.33$. Find the luminosity of this star in terms of the Sun's luminosity.

RJP-15.0 Two stars are observed to have the same apparent visual magnitude V= 7.50. Star 1 has $B_1=7.20$ while star 2 is observed to have $B_2 =8.70$. Assume no reddening.

A. Which star has the greater temperature?

B. Which star is intrinsically bluer and by how much?

RJP-16.0 A main sequence star is observed to have V=8.30, B = 8.00, and a parallax of 0.002".

A. Calculate the absolute visual magnitude, assuming no reddening.

B. Calculate the absolute bolometric magnitude.

C. Calculate the luminosity of the star in terms of the Sun's luminosity.

RJP-17.0 The star JG5 Ophiuchi has observed values of $m_v = 4.74$ and $m_B = 5.98$, and spectral type K2 III. Assume the standard reddening law and find

A. The color excess for this star

B. The distance of the star in parsecs

C. The radius of the star in terms of the Sun's radius.

RJP-18 The luminosity of a star is $L_* = 10^{40}$ ergs/sec and has a radius of 7.00×10^{13} cm.

A. Calculate the absolute bolometric magnitude of the star.

B. Calculate the surface temperature of the star.

RJP-19 A star has a BC = - 0.40 and $m_v =3.50$. Assume the star is not reddened.

A. Calculate $m_{bol}(*)$

B. Calculate $F_{bol}(r)$ if $m_v(\odot) = -26.70$ and $BC_\odot = -0.07$. Look up 2 other needed parameters for Sun.

RJP-20. The star ω Gem has $m_v = 7.16$ and spectral type G5 III. If $\Delta m_v = 2.00$ magnitudes as a result of interstellar reddening,

A. Find the distance modulus for this star.

B. Calculate the distance of the star in parsecs.

RJP-22. Use the spectroscopic parallax method to find the distance of a star that has a measured values of V=7.20, B= 6.99, and a spectral class of B2 V. Use interpolation to find $(B-V)_i$, if necessary.

RJP-30 A. Calculate the rms speed for He atoms at room temperature (T=300K) and for the surface temperature of the Sun.
B. Repeat the calculations for monatomic N.

RJP-31. Prove the validity of equation (2-29).

RJP-32 A. Calculate the monochromatic surface flux at λ = 4860Å for a star with a surface temperature of 5000K.
B. If the star has a radius of $5R_\odot$, what is the observed monochromatic flux from this star at a distance of 10 light years?

RJP-33. Write a FORTRAN program that computes the monochromatic flux at a given wavelength in nm for a given temperature as given by Planck's Law. Run the program for T= 300K , 1000K, and 2000K. Choose an appropriate wavelength range and step in wavelength. Export the data to a plotting program, and plot the results. Put all 3 curves on the same graph.

RJP-34. Derive Wien's Displacement Law from Planck's Law.

RJP-35. A star has a spectrum where λ_{max} is 2500Å. Calculate the surface temperature of the star.

RJP-36. Modify the program for RJP-33 to do a numerical integration in a loop of the area under a Planckian curve between any 2 wavelengths for a given temperature. The area is to be approximated by a finite sum of the differential areas in rectangles of height B_λ and base width $\Delta\lambda$. Then run the program using a value of $\Delta\lambda$ = 5nm from 200nm, to 1500nm for each of the temperatures in the previous problem and 3000, 6000, and 12000K. Plot your results versus temperature along the abscissa. Fit this curve with a polynomial. What should it be?

RJP-41. Calculate the wavelength in Å for the following: (a) the Lyman limit, (b) Lα.

RJP-42. Calculate the wavelength in Å for the Paschen limit.

RJP-51 A). What is the rms speed of an electron with just sufficient energy to ionize an unexcited sodium atom by collision? The ionization potential for Na I is 5.138ev.
B). What is the corresponding speed of a proton to do the same.
C). What is the temperature for an electron gas where the rms speed of the particles corresponds to the speed in part a?

RJP-52. Starting with the Maxwell-Boltzmann speed distribution function show that the most probable speed, v_p, is $(2kT/m)^{1/2}$.

RJP-53 Find the ratio N_2/N_1 for hydrogen at a temperature of 10,000K.

RJP-54 Calculate the temperature at which the n=2 level in H is populated the same as the n=1 level.

RJP-55 Calculate the ratio of the number of HI atoms in the first excited state to the number in the ground state for a G2 V star and compare with the same ratio for an A0 V star

RJP-56 Find the ratio of the strengths of the H_α and H_β lines for T=8000K, assuming the line strengths depend only on the relative populations of the relevant energy levels. Neglect cascading effects.

RJP-57 Calculate the partition function for H I at 6000K, summing over (a) only the ground state, (b) only the ground state and the first excited state, and (c) only for n=1 to 3. Does the series converge? Explain.

RJP-58 Calculate the number of H I atoms in the first excited state if T=6000K, the total density is of the gas is $2\text{x}10^{-7}$gm/cm^3, $N_o/N_+ = 0.05$, and the composition is 72% H and 28% He.

RJP-61. Prove that the ratio of the number of hydrogen atoms in the n=2 level to the total number of Hydrogen atoms, $[N_2/(N_o+N_+)]$ is very nearly approximated by N_2/N_1 at a temperature of 10,000K.

RJP-65 Find the number of H I atoms in the n=2 state for a gas consisting only of H atoms with a mass density $\rho = 1\ \text{x}10^{-7}$ g/cm^3 and T= 10,000K.

RJP-67. Calculate the relative line strengths of Fe II to Fe I lines for the star Sirius. Assume the electron density is 10^{14}, B for Fe I = 31.62, and B for Fe II = 19.95. Look up all other information needed and quote your source. Neglect oscillator strengths.

RJP-68 The surface temperature of a star is 15,000K and the photospheric density is 5.60e-10 gm/cm^3, 30% if which is helium, 70% is hydrogen, The ionization potential of HeI is 24.58 ev and the partition functions are $B_{HeI}=1$ and $B_{HeII} = 2$. If the electron density in the photosphere is 2.64e14 cm^{-3}, use the Saha equation to find the number density of HeII ions. This is a bit challenging; you need to solve two equations in two unknowns.

RJP-70 The observed continuum flux near a particular spectral line is $5.000 \text{ x } 10^{-10}$ ergs/cm^2 /sec/Å, and the flux at the bottom of the line is $4.167 \text{ x } 10^{-10}$ ergs/cm^2 /sec /Angs. What is the optical depth in the atmosphere of the star for the radiation at the bottom of the line relative to the continuum?

RJP-72 The number density of atoms in a certain layer of the atmosphere of a star is given by $n=n_oe^{-br}$, where $b = 4.55 \text{ x } 10^{-4}$, $n_o=3.56 \text{ x } 10^{13}$ per cm^3, and the atomic cross-section for absorption is $2.52 \text{ x } 10^{-18}$ cm^2 at λ = 550mn. The layer is $8.24 \text{ x } 10^3$ cm deep. Assume the temperature at the bottom of the layer is 6000K and the emissivity of the layer is 0.0 Compute the emergent flux from the layer at 550nm, assuming $I_{o\lambda} = B_\lambda$ for the intensity at the bottom of the layer.

RJP-74 Assume a stellar atmospheric layer is homogeneous in j_λ, and α_λ at T=5800K. The radiation incident on the bottom of the layer is given by the Planck Law for T=6000K. Take $\alpha_\lambda = \sigma_\lambda n_o$ as given in problem 70. Compute the emergent flux at 550nm from the layer which has thickness $8.24 \text{ x } 10^3$ cm and assume the source function is the Planck Law.

RJP-76 A certain star has an effective temperature of 12,000K. The gray absorptivity of its atmosphere is given by $\alpha = \alpha_o\exp(bz)$, where $\alpha_o = 8.25\text{x}10^{-11}$cm^{-1} and b=$2.50\text{x}10^{-10}$ cm^{-1}. Use the Eddington Approximation to find the temperature at a depth of $2.00\text{x}10^9$cm.

RJP-80 A certain star has a photospheric surface at T= 6000K. The chromosphere of the star is an optically homogeneous layer of thickness L=1 at T=3000K with an absorptivity of

$$\alpha_\lambda = \alpha_o \exp-[(\lambda - 4500)_{abs}/10]^2,$$

where the units are for λ in Ångstroms, $\alpha_o = 1.22$, and $j_\lambda = 0$. Compute the emergent spectral flux in steps of 2.5 Å from 4000 Å to 5000 Å. Plot the results with an appropriate scale.

RJP-88 Write a Fortran program that computes an absorption line profile with a broadening function given by:

$$\phi_\lambda = (1/\Delta\lambda_D)\exp\{-[(\lambda-\lambda_o)/\Delta\lambda_D]^2\}$$

Take $\Delta\lambda_D$=4Å and λ_o = 5500Å. Calculate values for ϕ_λ every 0.05Å over the wavelength range from 5400 - 5600 Å. Assume a constant continuum flux, $F_\lambda = 5.00$ and a value for D=0.60, where the product $F_\lambda D\phi_\lambda$ is the amount of flux absorbed at wavelength λ from the continuum. Plot the observed or net flux versus λ over the above wavelength range. (1) What is the numerical relationship of the HWHM of the line profile to the value of $\Delta\lambda_D$? Solve the profile equation for the flux to be one half to find out. (2) What happens if you change D? Plot values of the profile for different values of D.

RJP-90 Spectrophotometric data for the He II line near 1640Å for the star Y Cygni is in the EXCEL file "Y Cygni HeII" which is to be found on this web site. The spectral interval in this file is from 1630 to 1650 Å and the flux has been multiplied by a factor of 1e10 erg/cm^2/sec/Å. Download this file and plot it. Then find the equivalent width of this line in the way that is described in the online document "Computing W."

RJP-92 A stellar atmosphere has a temperature of 28,000K. The equivalent width of the Silicon IV line at 1350Å is measured to be 1.5Å in most hot stars. Is this due to thermal broadening? Assume the turbulent velocity is zero.

RJP-94. Compute the thermal width of the H_β spectral line in a star with a temperature of 5800K. Assume the turbulent velocity is 0.

RJP-101 Find the value of N_a for the total number of sodium atoms in the Sun from the information given in the notes at the end of Chapter 4C. The electron pressure is 10 dynes/cm^2.

RJP-103 Compute the equivalent width of the line found from problem 80.

RJP-105 The equivalent width of the Ca II line at λ_o = 3968.468Å in a certain star is W=1.05Å. The transition probability for this line is f =0.633. Use the curve of growth given in the notes at the end of Chapter 4C to find the value of N_a for Ca II.

RJP-106 Use the result of problem 105 to find the total column density of all Ca atoms in the star's atmosphere (chromosphere). The spectral line above is for a transition from the ground state to the 2nd excited level. Take the photospheric temperature to be 10,000K, the electron pressure as 10 dynes/cm^2, and the ionization potential of Ca II is 6.11 ev, $g_1 = 2$, $g_3 = 18$, Z_I = 4.7, and Z_{II} = 3.55.

RJP-110 The optical depth for the shock front in the binary star EM Car has been modeled to be 0.049 for radiation scattered by C IV ions in a particular bandpass. The thickness of the shock is 2 $R_\odot$ and the atomic cross-section for scattering in this bandpass is 1.21e-22cm^2.

(A). Calculate the number density of C IV ions in the shock.
Relative to hydrogen, the carbon abundance is 3.60e-3 and the helium abundance is 0.100. That is, for every carbon atom there are 277.77 H atoms. Assume the shock consists only of H, He, and C and assume all the carbon in the shock exists in the form C IV. (B) Calculate the mass density of the shock. (C). If the C IV/C III ratio were 0.5, would the mass density be higher or lower than what was calculated in (B)?

RJP-115 Calculate the molecular weight for a layer in a star with a temperature of 20,000K, a density of 7.00 x10^{-6} g/cm^3 and a composition of 71.5% H, 27.8% He, 0.6%C, and 0.1%O by mass.

RJP-121 Calculate the central temperature for a star with a mass of $2M_\odot$ and radius $4R_\odot$ with X=0.70, Y=0.28, and Z=0.02.

RJP-123 A star has a surface temperature of 10,400K, a radius, $R_*=2.6\ R_\odot$, and a total mass of 3.6 $M_\odot$. Assume 0.15 of its mass is in the core and available for fusion and X=0.70.
(A) Calculate the amount of energy that is produced by hydrogen fusion during the star's main sequence lifetime. The mass deficit for hydrogen fusion is 0.0286 amu.
(B) Calculate the main sequence lifetime for this star.

RJP-126 Calculate the main sequence lifetime of an O9 star with $R=12.6R_\odot$, T=31623K, and $M=18.62M_\odot$. Assume 0.10 of the star's mass is available for H fusion in the core.

RJP-128 Calculate the luminosity of a main sequence star that is the result of the p-p chain if these reactions occur within a core of radius $0.300R_*$, where $R_* = 9.00 \text{ x } 10^{10}$ cm, and the temperature in the core is uniform at $15\text{x}10^6$K. Use the usual density law given in class (5-41) with a = 3.05 x 10^{-11} and ρ_{core}=160 g/cm^3, and X=0.70. Take ε_0 to be 1.80 x 10^{-32}.

RJP-130. Compute a model of a non-convective, K0 V star just before the onset of hydrogen fusion. Assume $\rho(r)$ as given in notes. Assume μ depends on the temperature of the layer by using the Saha equation and that the total luminosity of the star originates in the central layer. Use X=0.70, Y=0.28, and Z=0.02.

RJP-140 Construct a model for the Sun consisting of 10 layers, by computing the density, pressure, and temperature in each layer.

RJP-144 The mass of a star is 1.05 solar masses, has an effective temperature of 3e4 K, and a luminosity of $0.05L_\odot$, calculate the following: (A) the radius of the star, $R_\ast$, (B) the mean density of the star, (C) the central pressure of the star, (D) the central temperature of the star.

RJP-146 Derive an expression for the potential energy of a gravitationally bound system of total mass M that has a density function $\rho = \rho_c(1-r/R_*)$, where ρ_c is the central density.

RJP-151 As a result of collisional excitation, HI atoms in the interstellar medium radiate 21 cm radiation due to the flipping of the electron in the magnetic field of the nucleus. Calculate the energy difference between the two states of the electron. Assuming that 0.5 of this energy is imparted by each of the colliding atoms to the electron in the target atom, calculate the kinetic temperature of the gas.

RJP-154 Assume the mass of the Sun were evenly distributed in a spherically symmetric cloud with a radius equal to the known radius of the solar system. What would the temperature of this cloud have to be in order to be stable against spontaneous gravitational collapse?

RJP-156 Compute the total thermal energy in the Sun just before the onset of hydrogen fusion. Assume the Sun collapsed from a very large cloud to its present size in accordance with the viral theorem, just before TNF initiated.

RJP-160 Calculate the Jeans length for an interstellar cloud with a temperature of 100K, a uniform density of $n = 5.0 \times 10^{10}$ cm^{-3}, and a composition of $X=0.34$, $Y=0.63$, and $Z=0.02$.

RJP-165. What must be the critical radius of an inhomogeneity that develops in the above cloud for fragmentation to occur if the density in the homogeneity were increased by a factor of 10^3 from the original density.

#170. The core temperature of a star is 12.0×10^6 K and the mass of the star is $1.22M_\odot$. If the mass in the core is 0.80 of the total mass, what must be the radius of the core for it to be completely degenerate?

APPENDIX

Physical constants:

Atomic mass unit (amu) = 1.66044×10^{-24} g.
Boltzmann constant, $k = 1.384 \times 10^{-16}$ erg/K = 8.61×10^{-5} ev/K
Electron mass, $m_e = 9.1091 \times 10^{-28}$ g
Hydrogen mass, $m_H = 1.67343 \times 10^{-24}$ g (includes mass of electron and binding energy)
Gravitational constant = 6.67×10^{-8} dynes cm^2/g^2
Planck's constant, $h = 6.6256 \times 10^{-27}$ erg sec
Proton mass $m_p = 1.67252 \times 10^{-24}$ g
Solar mass $M_\odot = 1.989 \times 10^{33}$ g
Solar radius $R_\odot = 6.96 \times 10^{10}$ cm
Solar luminosity, $L_\odot = 3.89 \times 10^{33}$ erg/s
Speed of light, $c = 2.99 \times 10^{10}$ cm/s
Stefan-Boltzmann constant, $\sigma = 5.6697 \times 10^{-5}$ erg cm^{-2} sec^{-1} K^{-4}
1 joule = 10^7 ergs
1 ev = 1.602×10^{-12} ergs = 1.602×10^{-19} j